Prague 1 January 2023
Ref. ČTÚ-55 278/2022-622

Within the scope of its competence, the Czech Telecommunication Office (hereinafter referred to as the "Office") performs measurement and evaluation of data parameters of electronic communications networks. Evaluation of the impact of the capacity of the electronic communications networks on the performance of internet access services is specified in

Methodology for assessment of the impact of the capacity of the electronic communications networks on the performance of internet access services, version 1.0, which is published and applied by ČTÚ/CTU as part of its inspection activities.

The evaluation of the impact of the capacity of electronic communications networks, namely in the segment of the electronic communications distribution or access network, on the performance of internet access services is based on the queueing theory, assuming that the elementary input data flow of requests on the queueing is stationary, regular and independent in terms of increments.

I. Introduction

The purpose of this document (hereinafter 'Methodology') is to describe and harmonise the procedure for evaluating the impact of the capacity of electronic communications networks, namely in the segment of the electronic communications distribution or access network, on the performance of internet access services or the impact on the actually achieved speed (hereinafter 'AAS'). The methodology builds on the Methodology for Measurement and Evaluation of Data Parameters of Fixed Electronic Communications Networks (hereinafter "Methodology for measurement") and the Methodology for Measurement and Evaluation of Data Parameters of Mobile Electronic Communications Networks, while respecting the General Authorisation No. VO-S/1/08.2020-9 laying down the conditions for the provision of electronic communications services. This Methodology is in line with BEREC Guidelines BoR (22) 81: *Implementation of the Open Internet Regulation* and its essence is based on the queueing theory with the general assumption that the elementary input flow of requests constitutes the Poisson process (on which the Erlang's formulae are based). The elementary input flow, i.e. the data flow in the case of electronic communications networks, is generally characterised as input data requests that meet the following characteristics, i.e.:

- *stationarity*: number of data flows (network termination points; hereinafter "NTP") which come to the queueing system over time Δt depends only on the length of that interval and does not depend on its position on the timeline,
- *regularity*: the probability of more than one data flow occurring in a sufficiently short interval of length Δt is negligibly low;

- *independence of increments*: the number of data flows occurring in a single time interval does not depend on the number of data flows in other intervals.

II. Queueing Theory

Due to the stationarity of the Poisson process, the number of data flows[1] does not depend on time t_0 but depends only on the length of the time interval Δt under consideration. If $p_k(t)$ indicates the probability that there are k data flows in an electronic communications network (hereinafter "network") over time t, the regularity of the Poisson process determines the probability that there will be k data flows over time in the network, whether it is a distribution or an access network, which is simultaneously equal to the probability that there were $(k-1)$ data flows at time t in a network and that during a time interval Δt, one data flow entered the network with probability $\lambda\Delta t$ or that there were k data flows in the network at time t and no new data flow entered the network during the time interval Δt with probability $(1-\lambda\Delta t)$. The following relationship will therefore apply:

$$p_k(t+\Delta t) = p_{k-1}(t) \cdot \lambda\Delta t + p_k(t) \cdot (1-\lambda\Delta t); k = 1,2,\dots \tag{1}$$

The probability that there is no data in the network at time $(t+\Delta t)$ is given by the probability that there was no data flow there and no data flow entered during the time interval Δt, i.e.:

$$p_0(t+\Delta t) = p_0(t) \cdot (1-\lambda\Delta t). \tag{2}$$

Equation (1) can be adjusted in a simple way to form equation (3), as well as equation (2) can be adjusted to the form of equation (4):

$$\frac{p_k(t+\Delta t)-p_k(t)}{\Delta t} = \lambda p_{k-1}(t) - \lambda p_k(t); k = 1,2,\dots, \tag{3}$$

$$\frac{p_0(t+\Delta t)-p_0(t)}{\Delta t} = -\lambda p_0(t). \tag{4}$$

If the time interval Δt is by limit close to 0, i.e. $\Delta t \to 0$,the left sides of equation (3) and of the equation (4) will be equal to the derivative of functions $p_k(t)$ and $p_0(t)$ at time t , respectively $p_k{}'(t)$ and $p_0{}'(t)$, while the limit transition does not have any effect on the right sides of equation (3) and of equation (4). It is possible to write down:

$$p_k{}'(t) = \lambda p_{k-1}(t) - \lambda p_k(t); k = 1,2,\dots, \lambda > 0, t > 0 \tag{5}$$

$$p_0{}'(t) = -\lambda p_0(t). \tag{6}$$

The equations (5) and (6) represent a system of first-order differential equations. To solve them, it is necessary to know the initial entry conditions. However, there are no data flows at time $t = 0$ in the network yet and therefore, the following applies:

$$p_k(0) = 0; k = 1,2,\dots, \tag{7}$$

$$p_0(0) = 1. \tag{8}$$

The solution of a system of the differential equations (5) and (6) with initial conditions defined in the form (7) and (8) is the function:

$$p_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t};\ k = 0,1,2,\dots,\lambda > 0, t > 0 \tag{9}$$

which represents the probability that k data flows (NTP) will come to a queueing system in time t. Thus, the elementary input data flow corresponds to a random quantity which has Poisson

[1] Within the Methodology, the term "data stream" refers to a set of all data transfers (information) of different nature and number of TCP connections that are related to one NTP at a given time. It is a general premise that one NTP corresponds to one available connection or access independently of the DeP.

probability distribution with a parameter λt. The mean value of the random quantity is therefore λt and especially at time $t = 1$, the mean value of the random quantity is the average number of NTP which generate the data flow coming to the queueing system per unit of time, equal to the parameter λ. The parameter λ refers to the mean input intensity and thus expresses the average number of data flows (NTP) that have entered the queueing system per unit of time.

The function (9) can be rewritten for a special case, where $t = 1$:

$$p_k(1) = \frac{(\lambda)^k}{k!} e^{-\lambda};\ k = 0,1,2, \dots, \lambda > 0. \tag{10}$$

If we consider another special case in function (9), when $k = 0$, we find out that the gaps between the arrivals of individual average numbers of data flows (NTP) to the queueing system are subject to exponential distribution. In this case, a probability, which equals $p_0(t)$, applies that after entering of one average number of data flow (NTP) no additional requirement has entered the queueing system for the entire interval t and the function (9) can be adjusted to an equation:

$$P(T > t) = p_0(t) = e^{-\lambda t};\ \lambda > 0, t > 0 \tag{11}$$

based on which we get the known distribution function $F(t)$ of exponential distribution:

$$F(t) = P(T \leq t) = 1 - P(T > t) = 1 - e^{-\lambda t};\ \lambda > 0, t > 0. \tag{12}$$

III. Impact of network capacity

The network capacity, be it a *backhaul network* or *distribution network,* significantly affects the performance of the provided internet access services. Insufficient network capacity causes an increase (degradation) of QoS parameters, which fall into a set of extended data parameters according to the Methodology for measurement,[2] respectively, frame (packets) time delays FTD, variations in inter-frame (packets) delay variation IFDV and frame (packets) loss ratio FLR, including IP packet error ratio IPER. Since the performance of internet access services is characterised by a AAS value that can be imagined as a transmission speed corresponding to the transport layer of the ISO/OSI model (L 4) when using the connection-oriented TCP protocol or as a volume of data payload transmitted over a given time interval, it is precisely the link orientation of the TCP protocol or the self-regulatory algorithm *TCP Congestion Control* what causes a decrease in the resulting AAS value over the observed time interval, depending on the increase in QoS parameters. The resulting AAS value in a specific NTP (depending on DeP) can be determined by means of the measurement process specified in the Methodology for measurement,[2] however, for the global assessment of the impact of network capacity, depending on the number of NTP, on the resulting decrease in the performance of internet access services (AAS), it is necessary to apply this Methodology, which is, in addition, designed to be used also for activation, project-related, or similar activity.

End-users use the internet access service randomly and independently on the basis of equal probability. The probability of end-users using the internet access service at the same time has Poisson distribution. It is presupposed that the number of end-users generating data flow over time t corresponds to the number of NTP[1] and this number is generally referred to as N. The probability that a number of N data flows (NTP) enters the queueing system can be written down in the form of a distribution function:

$$P(k \leq N) = P(k = 0) + P(k = 1) + \cdots + P(k = N) = \sum_{k=0}^{N} p_k(t), \tag{13}$$

and if we add the probability function (10) describing a special case when $t = 1$ into the equation (13), the resulting probability can be written down in the form:

[2] Czech Telecommunication Office, Prague, 2021: *Methodology for measurement and evaluation of data parameters of fixed electronic communications networks 2.1*

$$P(k \leq N) = \sum_{k=0}^{N} \frac{(\lambda)^k}{k!} e^{-\lambda};\ \lambda > 0. \qquad (14)$$

The resulting equation (14) for a distribution function describing the probability that N data flows generated by the same number of NTP will come into the queueing system can be used to assess the impact of network capacity on the performance of the internet access service. It is also appropriate to take into account the behaviour of end-users in a given location, which is expressed through UF *(utilisation factor*) parameter based on the results of network traffic monitoring.

1. Number of network termination points (NTP)

When assessing the impact of network capacity on the average number of NTP that will be able to provide the estimated limit value of the service performance, i.e. the AAS transmission speed, where, for example, from the point of view of the provision of internet access services at a fixed location, the limit value will be the normally available speed (‘the’NAS), i.e. a situation in which $AAS = NAS$, it is necessary, first of all, to rely on the actual knowledge of the network capacity value . The initial assumption is a situation where the NTP interface (depending on the DeP type) and the interface of the assessed connection or distribution network complies with the common IEEE 802.3 standards. The network capacity is most often determined by the bitrate value NBR corresponding to the physical layer of the ISO/OSI model (L 1), or by the IR speed corresponding to the connection layer of the ISO/OSI model (L 2) or by the IP throughput value $IP\ TR$, i.e. the speed corresponding to the network layer of the ISO/OSI model (L 3). The actual network capacity value can be verified by measuring the bandwidth in the downlink or uplink direction using a method specified in the measurement methodology,[3] while during the measurement process, the non-connection oriented UDP protocol is assumed to be on the transport layer. The measurement methodology[3] also makes it possible to assess the impact of network capacity on the basis of the result of network traffic monitoring, given its monthly timeframe. In order to assess the impact of network capacity under this methodology, it is necessary to first recalculate the network capacity value to the transport layer of the ISO/OSI model (L 4) corresponding to the normal TCP protocol header value (e.g. 20 B).

An example could be the IEEE 802.3z interface, where the achieved bit rate on the physical layer of the ISO/OSI model is $NBR = 1000\ Mb/s$. The maximum available information rate IR on the connection layer of the ISO/OSI model is limited by the maximum number of FPS frames:

$$FPS = \frac{NBR}{(IFG+Preamble+MAC\ DST+MAC\ SRC+Ethertype+802.1Q(802.1ad)+MTU+FCS)\cdot 8}\ [1/s; b/s, B], \quad (15)$$

where in the example given, it is assumed that values $IFG\ 12\ B, Preamble = 8\ B, MAC\ DST = \\ = 6\ B, MAC\ SRC = 6\ B,\ 802.1Q\ (802.1ad), = 0\ B, Ethertype = 2\ B,\ MTU = 1500\ B$ and $FCS = 4\ B$. In such a case, the interface corresponding to the IEEE 802.3z standard will, according to the relationship (15), reach the value $FPS = 81274\ 1/s$. The value of the resulting TCP throughput $TCP\ TR$, i.e. the speed corresponding to the transport layer of the ISO/OSI model when using the TCP protocol, where the IPv4 protocol without the optional parts of the header (20 B) and TCP header without any extension (20 B) are used on the network layer, is determined according to the equation:

$$TCP\ TR = (MTU - IP_{header} - TCP_{header}) \cdot 8 \cdot FPS; [b/s; B, 1/s]. \qquad (16)$$

In the example given, the interface corresponding to IEEE 802.3z standard achieves the TCP throughput value $TCP\ TR = 949{,}285\ Mb/s$. If IPv6 protocol is used, for reasons of greater header (40 B), the TCP throughput $TCP\ TR$ value is lower, namely 936.28 Mb/s. For simplicity, only IPv4 will be further considered, but the calculations can be adjusted similarly for IPv6.

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[3] Huawei, 2016: *WTTx Capacity White Paper*

Knowing the value of the network capacity on the transport layer, it is further necessary to establish the limit value of the transmission speed (AAS) assessed in terms of the impact of the network capacity on the total number of NTP which will achieve the set limit transmission speed value with a probability corresponding to the distribution function according to the relationship (14). An example could be a deployed *access network* corresponding to ITU-T G.984 standard, or deployed by *gigabit passive optical networks* (GPON), and following the above recalculation between ISO/OSI model layers, it is assumed that GPON *optical line termination* device is connected to the distribution network via *small form-factor pluggable (SFP)* module for a fiber-optic cable corresponding to IEEE 802.3z standard, e.g. in the form of a 1000BASE-SX variant. In the considered example of the provision of an internet access service at a fixed location by GPON technology, the value corresponds to $\mathrm{NAS} = 60\ \mathrm{Mb}/s$ in the download direction. In order to simplify the calculation of the impact of network capacity, it can therefore be stated that $\mathrm{AAS_{min}} = \mathrm{NAS} = 60\ \mathrm{Mb/s}$. To ensure that the $\mathrm{AAS_{min}}$ value is not lower than NAS the average number of NTP generating data flow over time $\mathrm{t} = 1$ shall not be higher than the defined number N, where the CAP parameter denotes the network capacity corresponding by its value to the transport layer (TCP protocol):

$$\mathrm{N} = \left\lfloor \frac{\mathrm{CAP}}{\mathrm{AAS_{min}}} \right\rfloor = \left\lfloor \frac{949{,}285}{60} \right\rfloor = 15\ [-;\mathrm{Mb/s},\mathrm{Mb/s}]. \quad (17)$$

It is then necessary to determine the probability value $\mathrm{P(k \leq N)}$ at which it will be ensured that the $\mathrm{AAS_{min}}$ value does not fall below the given NAS value. It is generally recommended that the probability value is not lower than 90%. For the purposes of the example, the lower limit of the limit value is selected as $\mathrm{P(k \leq N)} = 90\%$. The calculation of the mean input intensity value λ, which is the key parameter for determining the resulting average number of NTP, shall be carried out using the following relationship (14):

$$\mathrm{P(k \leq N)} = \sum_{\mathrm{k=0}}^{\mathrm{N}} \frac{(\lambda)^{\mathrm{k}}}{\mathrm{k!}} \mathrm{e}^{-\lambda} \geq 90\% \rightarrow \mathrm{P(k \leq 15)} = \sum_{\mathrm{k=0}}^{15} \frac{(\lambda)^{\mathrm{k}}}{\mathrm{k!}} \mathrm{e}^{-\lambda} \geq 0{,}9 \rightarrow$$
$$\rightarrow \lambda \doteq 11{,}140\ [-]. \quad (18)$$

The resulting mean input intensity value λ corresponds to the average number of NTP generating data flows over time t that will use the internet access service with 90% probability according to the Poisson process. In this case, the performance of the internet access service can therefore be defined as $\mathrm{AAS_{max}}$:

$$\mathrm{AAS_{max}} = \frac{\mathrm{CAP}}{\lambda} = \frac{949{,}285}{11{,}140} \doteq 85{,}214\ [\mathrm{Mb/s};\mathrm{Mb/s},-]. \quad (19)$$

The ratio of the average number of NTP and of the mean input intensity λ can be determined based on the relationship (17) and the relationship (19) as:

$$\frac{\mathrm{NTP}}{\lambda} = \frac{\frac{\mathrm{CAP}}{\mathrm{AAS_{min}}}}{\frac{\mathrm{CAP}}{\mathrm{AAS_{max}}}} = \frac{\mathrm{CAP}}{\mathrm{AAS_{min}}} \cdot \frac{\mathrm{AAS_{max}}}{\mathrm{CAP}} = \frac{\mathrm{AAS_{max}}}{\mathrm{AAS_{min}}} = \frac{85{,}214}{60} \doteq 1{,}420. \quad (20)$$

However, the value $\mathrm{AAS_{max}}$ may achieve by limit the speed of the corresponding network capacity. In the case of equal values of $\mathrm{AAS_{max}}$ and CAP capacity, it can be concluded that the average number of NTP corresponds to the relationship (21), where for simplification, the value $\mathrm{AAS_{min}}$ is generally referred to as AAS ($\mathrm{AAS_{min}} \rightarrow \mathrm{AAS}$):

$$\mathrm{AAS_{max}} = \mathrm{CAP} \implies \frac{\mathrm{NTP}}{\lambda} = \frac{\mathrm{CAP}}{\mathrm{AAS}} \implies \mathrm{NTP} = \left\lfloor \frac{\mathrm{CAP} \cdot \lambda}{\mathrm{AAS}} \right\rfloor\ [-;\mathrm{Mb/s},-,\mathrm{Mb/s}]. \quad (21)$$

The resulting relationship (21) for the calculation of the average number of NTP corresponds to the procedure for designating capacity dimensioning of distribution (connection) networks,[3] as indicated by telecommunication equipment manufacturers. A situation where the value $\mathrm{AAS_{max}}$ corresponds to the value of the network capacity will be covered in the Methodology section, which deals with the impact of UF *(utilisation factor*) parameter on the average number of NTP. The resulting average number of NTP value in the above mentioned example of GPON technology, where there is a probability $\mathrm{P(k \leq N)} = 90\%$ to ensure that if they simultaneously generate data flow in a queueing system, the AAS v does not fall below the selected value of $60\ \mathrm{Mb/s}$, is then determined on the basis of the relationship (21):

$$\mathrm{NTP} = \left\lfloor \frac{\mathrm{CAP}\cdot\lambda}{\mathrm{AAS}} \right\rfloor = \left\lfloor \frac{949{,}285\cdot 11{,}140}{60} \right\rfloor = 176\ [-; \mathrm{Mb/s}, -, \mathrm{Mb/s}]. \tag{22}$$

Thus, GPON technology with a dedicated network capacity of $1\ \mathrm{Gb/s}$ (L 1) at the input of OLT devices can provide an average transmission speed of $60\ \mathrm{Mb/s}$ for 176 TTE (*Telecommunications Terminal Equipment*), respectively *ONT (Optical Network Terminal*), with 90% probability. In the case of PON technology, it is recommended to configure the passive optical/fibre network infrastructure to a lower possible dividing ratio, in this case the ratio would be $1:128$. Similarly, the provision of an internet access service at a fixed location can also be done through mobile networks, respectively FWA technology. An example can be a 5G network technology in the $3{,}5\ \mathrm{GHz}$ band with the system bandwidth $\mathrm{B} = 100\ \mathrm{MHz}$, time slot configuration ratio between downlink and uplink $3:1$, while gNodeB corresponds to 64T64R and shows a sufficient level of EIRP on the terminal equipment side (e.g. $27\ \mathrm{dBm}$; 2T4R). The planning of the access network capacity to ensure sufficient connectivity of the mobile network (gNodeB), is based on dimensioning cell capacity in the downlink direction, considering the physical principles of behaviour of the mobile network technology. It is necessary to determine the limiting parameter value of the capacity expansion $\mathrm{CE_{thr}}$ (*capacity expansion threshold*) , as well as the time slot ratio value in the appropriate direction $\mathrm{TR_{DL/UL}}$ (downlink/uplink time ratio) and η (*spectrum efficiency*). The resulting downlink capacity of FWA technology is defined as:

$$\mathrm{CAP} = \mathrm{B}\cdot\mathrm{CE_{thr}}\cdot\mathrm{TR_{DL}}\cdot\eta[\mathrm{Mb/s}; \mathrm{MHz}, -, -, \mathrm{b/s/Hz}] \tag{23}$$

and for the example of 5G network technology operating in the $3{,}5\ \mathrm{GHz}$ band, parameters can be determined as $\mathrm{B} = 100\ \mathrm{MHz}$, $\mathrm{CE_{thr}} = 70\%$, $\mathrm{TR_{DL}} = 0{,}75$ and $\eta = 17\ \mathrm{b/s/Hz}$. Putting this into the equation (23), we get the available capacity $892{,}5\ \mathrm{Mb/s}$ (L 3). Since this value corresponds to the network layer of the ISO/OSI model, i.e. IP throughput $\mathrm{IP\ TR}$, it is necessary to convert it again to the transport layer using the TCP protocol, where the size of the TCP header without any extension (20 B) is assumed on the basis of adjusting the relationship (16):

$$\mathrm{TCP\ TR} = \mathrm{IP\ TR} - \frac{\mathrm{IP\ TR}\cdot\mathrm{TCP_{header}}}{\mathrm{MTU}-\mathrm{IP_{header}}}\ [\mathrm{b/s}; \mathrm{b/s}, \mathrm{B}, \mathrm{B}, \mathrm{B}]. \tag{24}$$

In this example, the capacity of 5G network technology operating in the $3{,}5\ \mathrm{GHz}$ band achieves the resulting TCP throughput $\mathrm{TCP\ TR} = 880{,}439\ \mathrm{Mb/s}$. As in the example of GPON technology, it is necessary to determine the limit value of the transmission speed, assessed in terms of the impact of network capacity on the average number of NTP, which will achieve the determined value of the transmission speed with probability according to the relation (14). This example will be based on the same value $\mathrm{NAS} = 60\ \mathrm{Mb/s}$. To ensure that the value of $\mathrm{AAS_{min}}$ is not lower than NAS, the number of NTP generating data stream for time $\mathrm{t} = 1$ shall be lower than the defined number N, where the CAP parameter indicates the designated network capacity at transport layer level (L 4):

$$\mathrm{N} = \left\lfloor \frac{\mathrm{CAP}}{\mathrm{AAS_{min}}} \right\rfloor = \left\lfloor \frac{880{,}439}{60} \right\rfloor = 14\ [-; \mathrm{Mb/s}, \mathrm{Mb/s}]. \tag{25}$$

It is then necessary to determine the probability value $\mathrm{P}(\mathrm{k} \leq \mathrm{N})$ at which it will be ensured that the AAS value does not fall below the specified NAS value. As in the example of GPON technology, the probability limit value $\mathrm{P}(\mathrm{k} \leq \mathrm{N}) = 90\%$ is selected for the purposes of this case. The calculation of the average number of NTP corresponding to the mean input intensity λ, which is the key parameter for determining the resulting average number of NTP, shall be performed using the relationship (14) stated above:

$$\mathrm{P}(\mathrm{k} \leq \mathrm{N}) = \sum_{\mathrm{k}=0}^{\mathrm{N}} \frac{(\lambda)^{\mathrm{k}}}{\mathrm{k}!} \mathrm{e}^{-\lambda} \geq 90\% \rightarrow \mathrm{P}(\mathrm{k} \leq 14) = \sum_{\mathrm{k}=0}^{14} \frac{(\lambda)^{\mathrm{k}}}{\mathrm{k}!} \mathrm{e}^{-\lambda} \geq 0{,}9 \rightarrow$$

$$\rightarrow \lambda \doteq 10{,}305\ [-]. \tag{26}$$

The resulting value of the average number of NTP for which it will be ensured with the probability $P(k \leq N) = 90\%$ that if they generate data flow simultaneously in a queueing system implemented by the 5G network technology in the $3{,}5\ \mathrm{GHz}$ band, the AAS value does not fall below the selected NAS value, is determined on the basis of the relationship (21):

$$\mathrm{NTP} = \left\lfloor \frac{\mathrm{CAP} \cdot \lambda}{\mathrm{AAS}} \right\rfloor = \left\lfloor \frac{880{,}439 \cdot 10{,}305}{60} \right\rfloor = 151\ [-; \mathrm{Mb/s}, -, \mathrm{Mb/s}]. \tag{27}$$

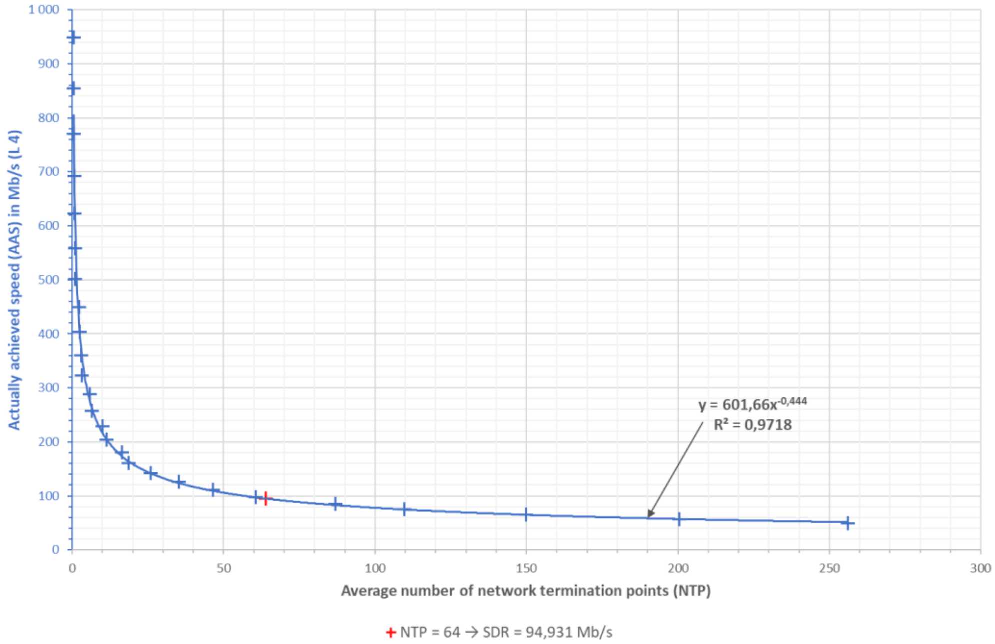

Fig. 1: The dependency of the actually achieved speed (AAS) on the average number of network termination points (NTP), example of GPON access technology with a backhaul capacity of 1000 Mbps (L 1)

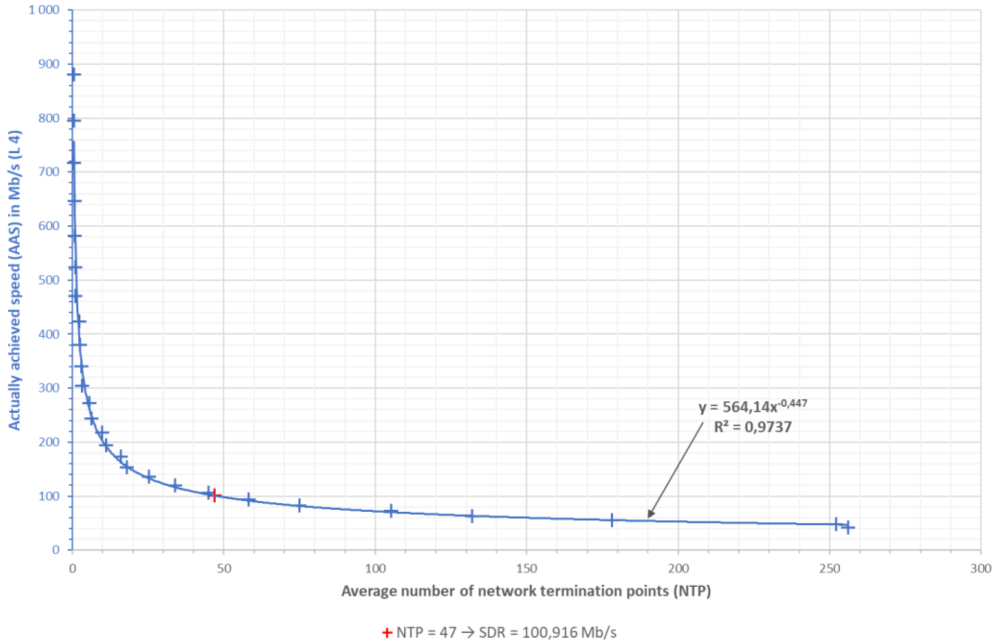

Fig. 2: The dependency of the actually achieved speed (AAS) on the average number of network termination points (NTP), example of FWA access technology (5G in $3{,}5\ \mathrm{GHz}$ band) with available backhaul capacity ≥ $892{,}5\ \mathrm{Mb/s}$ (L 3); without an assessment of the impact of MIMO

Thus, the example of an FWA technology with an access network capacity corresponding to at least the capacity determined on the basis of dimensioning cell capacity in the downlink direction (FWA technology will form a *bottleneck*) is capable to ensure, with regard to the physical principles of the behaviour of 5G network technology, i.e. $892{,}5\ \mathrm{Mb/s}$ (L 3), the average value of transmission speed of $60\,\mathrm{Mb/s}$, for 151 terminal equipment with 90% probability. The resulting impact of CAP network capacity on the average number of NTP that will be able to provide the estimated limit value of transmission speed or, from the point of view of the provision of internet access services at a fixed location, it will be a value of $\mathrm{AAS_{min}} = \mathrm{NAS}$, can also be displayed in the form of a conversion characteristic, for the specific value of the network capacity. The resulting conversion characteristic of the example of GPON technology with a network capacity of 1000 Mbps (L 1) is given in Fig. 1. In the displayed waveform of the conversion characteristic, it can be noted that in the case of average number of $\mathrm{NTP} = 64$, i.e. a typical basic dividing ratio of the $1:64$ passive fibre distribution network, the value is $\mathrm{AAS} = 94{,}931\,\mathrm{Mb/s}$ (red point in Fig. 1).

Similarly, the conversion characteristic is set for the example of the FWA access technology corresponding to the 5G network operated in the $3{,}5\,\mathrm{GHz}$ band ($\mathrm{B} = 100\,\mathrm{MHz}$) where the capacity of the backhaul corresponds at least to the downlink capacity of the cell and at least to the value $892{,}5\,\mathrm{Mb/s}$ (L 3), see Fig. 2. If this FWA access technology in the given configuration was to provide value $\mathrm{NAS} \geq 100\,\mathrm{Mb/s}$ for its terminal equipment at the same time, it would be able to do so with 90% probability for the average number of $\mathrm{NTP} = 47$ (red point in Fig. 2; without an assessment of the impact of MIMO).

2. Impact of the utilisation factor (UF)– network utilisation by end-users

One of the commonly used methods to identify the utilisation rate of the network, or network topology as a whole, is determination of the parameter of the use of individual data links LU (link utilisation) of the network topology. The LU parameter is defined as the ratio of nominal value of the speed over time (current bit rate NBR value) and of the corresponding capacity. This parameter can also be described as capacity utilisation and is always associated with a specific demarcation point DeP of the network topology. For an exemplar explanation of the impact of a LU parameter, or of utilisation factor UF , the factor of network utilisation by end-users, a sample of 4 providers of an internet access service with a connection capacity $\mathrm{CAP} = 1000\,\mathrm{Mb/s}$ is selected, where the results of network traffic monitoring of their backhauls will be an example of explanation. In the Fig. 3, daily monitoring of network traffic of the 4 selected backhauls (marks A, B, C and D) within the same day is displayed.

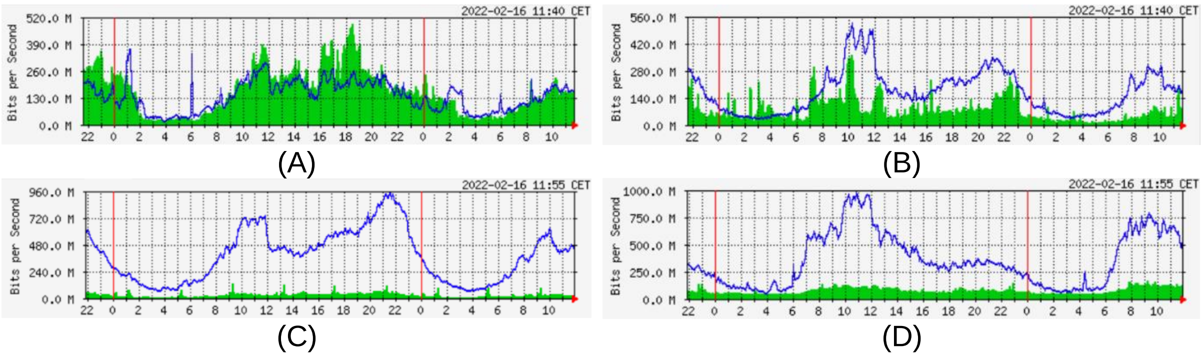

Fig. 3: Results of daily monitoring of network traffic of backhauls of selected sample of internet access service providers (A, B, C and D) over the same period of time, average values in 5 minutes interval

Then Fig. 4 displays the corresponding change in the value of the LU parameter at the time of the day (downlink direction marked in Fig. 3 with blue line). The value of the LU parameter depends on the time of the day and its waveform itself varies over time depending on the specific internet access service provider due to the typical behaviour of their end-users, given the type of data traffic and the specific location.

The dependence of the value of the data link utilisation parameter LU on the time of the day can also be expressed by the so-called relative value of the cumulative frequency, as shown in Fig. 5. The x-axis shows the levels of the data link utilisation LU, the y-axis represents the value of the share of the time of the day after which the data link was utilised by a specific level of LU parameter.

Tab. 1: Calculation of the utilisation factor UF for a selected sample of internet access service providers; the capacity of the link (backhaul) in all cases $1000\ \mathrm{Mb/s}$; the nominal speed value NBR at peak hours corresponds to the maximum waveform value in Fig. 4 (daily waveform)

Internet access service provider	Maximum bit rate $\mathrm{NBR_{max}}$	Average bit rate $\mathrm{NBR_{avg}}$	UF (*utilisation factor*)	Average LU (*link utilisation*)	timeshare/quotient of the day while $\mathrm{LU} \geq 0{,}5$
A	$365{,}63\ \mathrm{Mb/s}$	$159{,}15\ \mathrm{Mb/s}$	$0{,}435$	$0{,}159$	0%
B	$516{,}25\ \mathrm{Mb/s}$	$198{,}34\ \mathrm{Mb/s}$	$0{,}384$	$0{,}198$	6%
C	$960{,}00\ \mathrm{Mb/s}$	$450{,}93\ \mathrm{Mb/s}$	$0{,}470$	$0{,}451$	56%
D	$984{,}38\ \mathrm{Mb/s}$	$388{,}85\ \mathrm{Mb/s}$	$0{,}395$	$0{,}389$	42%

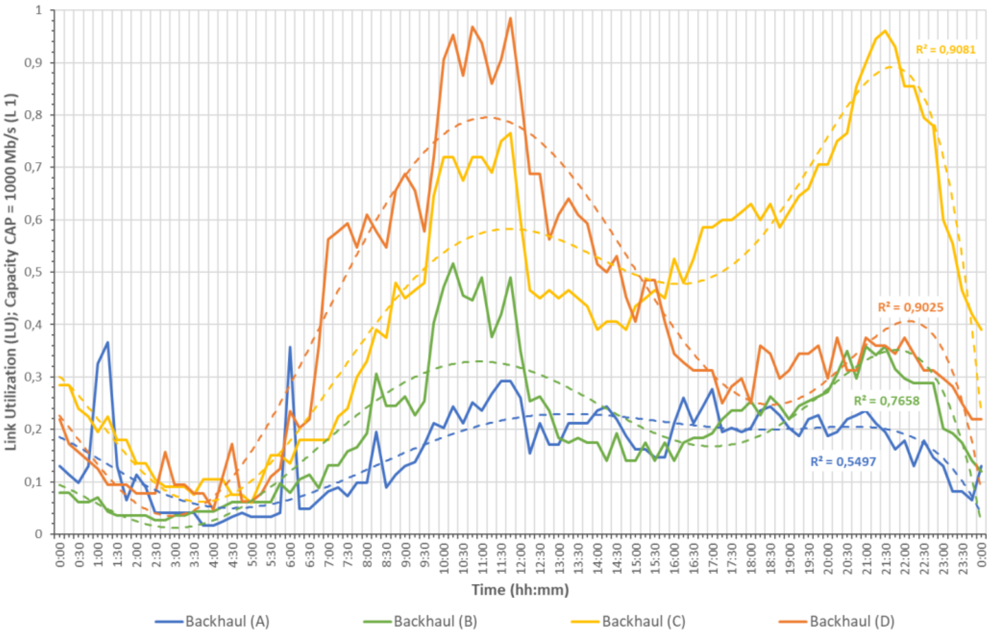

Fig. 4: The dependency of the value of the LU parameter of the data link utilisation on the time of the selected sample of internet access service providers; the capacity of the link (backhaul) in all cases $1000\ \mathrm{Mb/s}$; input data for analysis was based on network traffic monitoring with daily waveform (Fig. 3)

It follows from the waveform of relative values of the cumulative frequency shown in Fig. 5 that, for example, the backhaul of internet access service provider A reaches the value of the parameter $\mathrm{LU} \geq 0{,}4$ at approximately $6\ \%$ of time of the day, whereas the backhaul of the internet access service provider D reaches the value of the parameter $\mathrm{LU} \geq 0{,}4$ at approximately $68\ \%$ of time of the day. This method of interpreting the LU parameter makes it possible to obtain much more detailed information about the state of utilisation of the data link. The backhaul of the internet access service provider C reaches the value of the parameter $\mathrm{LU} \geq 0{,}5$ at approximately $56\ \%$ of time of the day. It is a common rule that if the average value of a LU parameter reaches a level of $0{,}4$ up to $0{,}5$, it is recommended to increase the capacity of the data link.

The character of the typical utilisation of an electronic communications network by end-users in a given location and by a type of traffic can also be described by means of an aggregate utilisation factor (UF), which is determined as the ratio of the average speed value (average bit rate) over a certain timeframe and of the nominal value of the bit rate at peak time. The timeframe refers to the result of network traffic monitoring, namely to daily, weekly or monthly waveform. The Office recommends assessing the parameter of utilisation factor UF based on the monthly waveform, according to the Methodology for measurement.[2] From the waveforms mentioned in Fig. 3 and Fig. 4, it is evident that the peak time is different for individual internet access service providers or in parts of their network topology. It is therefore recommended to use the maximum bit rate value NBR in the sequence timeframe as the nominal value of the speed at peak time. Calculation of utilisation factor UF based on the daily monitoring of network traffic (Fig. 3) is listed in Tab 1. It follows from the results shown in Tab. 1 that different access networks show similar values for the utilisation factor UF. The utilisation factor UF does not depend directly on the value of the capacity of the given data link (connected network), or on the current value of the utilisation parameter LU at a given time, but on the behaviour of end users depending on the location and the nature of the data traffic.

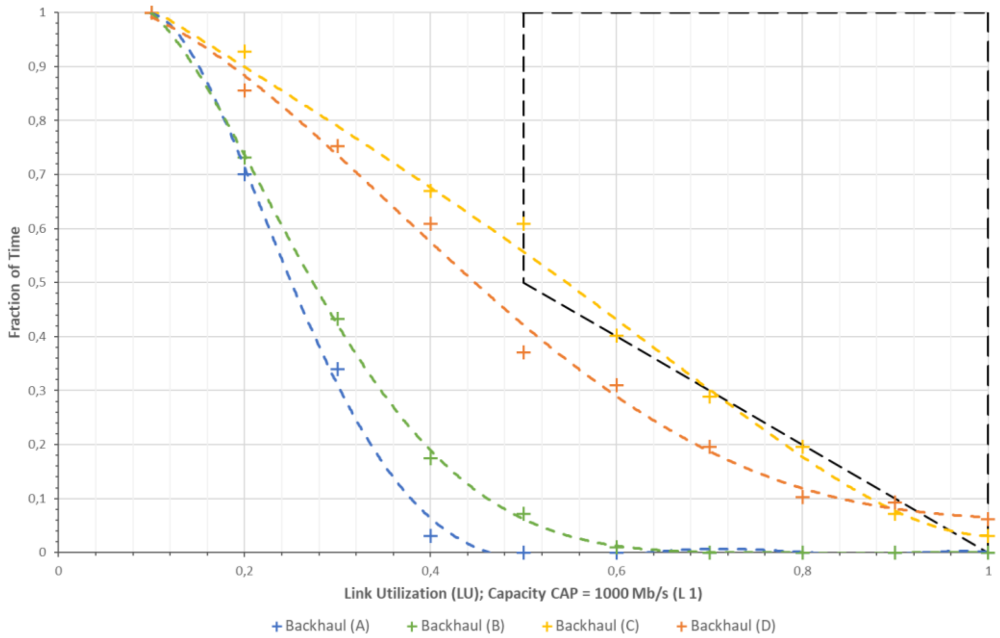

Fig. 5: The dependency of the relative value of the cumulative frequency of the data link utilisation parameter LU on the time of the day for a selected sample of internet access service providers; the capacity of the link (backhaul) in all cases $1000\ \mathrm{Mb/s}$; area of the LU parameter meaning strengthening the capacity bounded by black colour

Fig. 6 presents results of the monthly monitoring of the network traffic of the backhauls (downlink direction marked by a blue line) of a selected sample of internet access service providers (A, B, C and D), where the latest daily recording of the monitoring waveform shown in Fig. 6 correlates with the monitoring waveform stated above in Fig. 3. The results of calculation of the value of the utilisation factor UF based on monthly network traffic monitoring are given in Tab. 2.

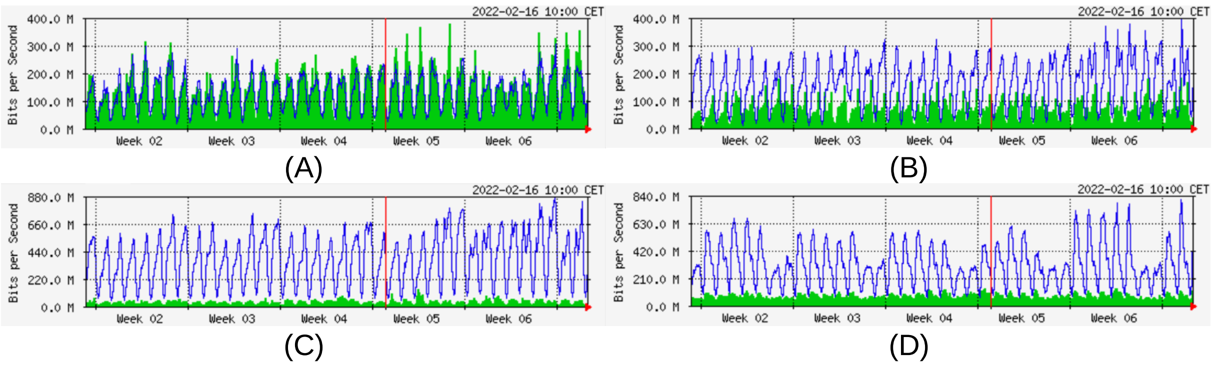

Fig. 6: Results of monthly monitoring of network traffic of backhauls of selected sample of internet access service providers (A, B, C and D) over the same period of time, average values in 2 hours interval

The results presented in Tab. 2 show that the percentage difference in the calculation of the value of the utilisation factor UF on the basis of the monthly (Fig. 6) and daily (Fig. 3) results of network traffic monitoring is around $6\ \%$ (average). This is due to different sampling in the case of monthly and daily network traffic monitoring (2 hours vs. 5 minutes). According to the Methodology for Measurement,[2] the Office recommends that the assessment shall be based primarily on the results of monthly network traffic monitoring, while the results listed in Tab. 2 show that in case of unavailability of monthly network traffic monitoring, it can also be proceed from the daily waveform.

Tab. 2: Calculation of the utilisation factor UF of a selected sample of internet access service providers; the capacity of the link(s) was in all cases $1000\ \mathrm{Mb/s}$; the nominal value NBR of the speed at peak hours corresponds to the maximum value of the waveform in Fig. 6 (monthly waveforms compared to daily waveform)

Internet access service provider	Maximum bit rate $\mathrm{NBR_{max}}$	Average bit rate $\mathrm{NBR_{avg}}$	UF (*utilisation factor*) monthly waveform	UF (*utilisation factor*) daily waveform	Percentage difference between daily and monthly waveform
A	302,2 Mb/s	127,3 Mb/s	0,421	0,435	3,32%
B	388,1 Mb/s	156,1 Mb/s	0,402	0,384	4,48%
C	870,1 Mb/s	366,5 Mb/s	0,421	0,470	11,63%
D	800,1 Mb/s	300,3 Mb/s	0,375	0,395	5,33%

The impact of the utilisation factor UF can be explained using the example of the result of the monthly network traffic monitoring of the part of the distribution network of the internet access service provider that shows the value of the average bit rate $\mathrm{NBR_{avg}} = 150\ \mathrm{Mb/s}$, where the part of the distribution network under consideration has a capacity of $1000\ \mathrm{Mb/s}$. At the value of the utilisation factor $\mathrm{UF} = 0{,}4$, the given part of the distribution network will reach bit rate $\mathrm{NBR_{max}} = 375\ \mathrm{Mb/s}$ ($\mathrm{LU_{max}} = 37{,}5\ \%$) at peak time, while at the value of the utilisation factor $\mathrm{UF} = 0{,}2$, it will reach a bit rate $\mathrm{NBR_{max}} = 750\ \mathrm{Mb/s}$ ($\mathrm{LU_{max}} = 75\ \%$), which means double utilisation of the part of the distribution network under consideration or of demarcation point DeP. It is therefore clear that the value of the utilisation factor UF has an indirect impact on the value of the average number of network termination points NTP that will be able to provide the expected limit value of the transmission speed or the NAS value. The calculation of the average number of NTP is based on the actual value of the network capacity, see the relationship (21). The assessment of the impact of the utilisation factor UF on the resulting average number of NTP, by which it will be ensured with a probability $\mathrm{P}(\mathrm{k} \le \mathrm{N})$ that if they generate a data flow simultaneously in the queueing system, the AAS value will not fall below the selected value, shall be determined on the basis of the presented relationship (21). On the basis of the knowledge of the value of the utilisation factor UF, the nominal value of the speed at peak time, i.e. the maximum bit rate $\mathrm{NBR_{max}}$, can be expressed as:

$$\mathrm{UF} = \frac{\mathrm{NBR_{avg}}}{\mathrm{NBR_{max}}} \Longrightarrow \mathrm{NBR_{max}} = \frac{\mathrm{NBR_{avg}}}{\mathrm{UF}}\ [\mathrm{Mb/s; Mb/s, -}]. \tag{28}$$

At the maximum bit rate $\mathrm{NBR_{max}}$, it is possible to express the data link capacity on the basis of the link utilisation $\mathrm{LU_{max}}$,if we assess this parameter at peak time (maximum values in the monitored time frame):

$$\mathrm{LU_{max}} = \frac{\mathrm{NBR_{max}}}{\mathrm{CAP}} \Longrightarrow \mathrm{CAP} = \frac{\mathrm{NBR_{max}}}{\mathrm{LU_{max}}} = \frac{\frac{\mathrm{NBR_{avg}}}{\mathrm{UF}}}{\mathrm{LU_{max}}} = \frac{\mathrm{NBR_{avg}}}{\mathrm{UF{\cdot}LU_{max}}}\ [\mathrm{Mb/s; Mb/s, -, -}]. \tag{29}$$

It is evident from the resulting relationship (29) that the utilisation factor UF has an impact on the resulting data link capacity, however, knowing the value of the utilisation factor UF is insufficient to assess the impact on the resulting value of the average number of NTP, as specified below. It is also necessary to know, according to the relationship (29), the average bit rate $\mathrm{NBR_{avg}}$. In a theoretical case with the maximum possible data link utilisation LU, where the maximum bit rate $\mathrm{NBR_{max}}$ equals to the link capacity CAP (in the case of backhaul A, $\mathrm{NBR_{max}}$ would equal 1000 Mb/s), the data link utilisation parameter LU shall equal to 1. This theoretical case exactly corresponds to the assumption given in relationship (21) for the calculation of the average number of NTP, i.e. the situation in which the link capacity is used as much as possible:

$$\mathrm{UF} = \frac{\mathrm{NBR_{avg}}}{\mathrm{NBR_{max}}} = \frac{\mathrm{NBR_{avg}}}{\mathrm{CAP}} \Longrightarrow \mathrm{NBR_{avg}} = \mathrm{UF} \cdot \mathrm{CAP} \tag{30}$$

and at the same time:

$$\mathrm{LU} = \frac{\mathrm{NBR}}{\mathrm{CAP}} \Longrightarrow \mathrm{LU_{max}}(\mathrm{NBR} = \mathrm{CAP}) = \frac{\mathrm{CAP}}{\mathrm{CAP}} = 1. \tag{31}$$

The impact of this combination of derived parameters according to relationship (30) and relationship (31) can be expressed using adjusted capacity CAP' as a theoretical change in the original capacity:

$$\mathrm{CAP}'_{\text{pro } \mathrm{LU_{max}}=1} = \frac{\mathrm{NBR_{avg}}}{\mathrm{UF} \cdot \mathrm{LU_{max}}} = \frac{\mathrm{UF} \cdot \mathrm{CAP}}{\mathrm{UF} \cdot \frac{\mathrm{CAP}}{\mathrm{NBR_{max}}}} = \frac{\mathrm{UF} \cdot \mathrm{CAP}}{\mathrm{UF} \cdot \frac{\mathrm{CAP}}{\mathrm{CAP}}} = \frac{\mathrm{UF} \cdot \mathrm{CAP}}{\mathrm{UF}} = \mathrm{CAP}. \tag{32}$$

The initial general relationship (21) to determine the value of the average number of NTP, for which it will be ensured with a probability $\mathrm{P}(\mathrm{k} \leq \mathrm{N})$ that if they generate a data flow simultaneously in the queueing system, the AAS value does not fall below the selected value, assumes a situation where a given data link is utilised by a bit rate NBR corresponding to the given link capacity, i.e. $\mathrm{NBR_{max}} = \mathrm{CAP}$. The value of the utilisation factor UF thus depends practically only on the average bit rate $\mathrm{NBR_{avg}}$ and will have no effect on the result of the initial relationship (21). A different situation occurs when a given data link is utilised by a bit rate NBR lower than the given link capacity, i.e. $\mathrm{NBR} < \mathrm{CAP}$. In this case, it is possible to take into account the value of the utilisation factor UF of the given data link, or the connection or distribution network (generally the network segment). If the impact of the utilisation factor UF is generally taken into account in the form of adjustment factor κ, we can write down the relationship between the original value of the link capacity CAP and the resulting value of the adjusted capacity CAP' in the form of a relationship:

$$\mathrm{CAP}' = \kappa \cdot \mathrm{CAP} \tag{33}$$

while in the case of the initial general relationship (21), it is evident that the value of the adjustment factor is $\kappa = 1$. An assessment of the impact of the utilisation factor UF on the value of the adjustment factor κ is carried out on the example of the backhaul A (Tab. 2) of the internet access service provider, by assessing the impact of the change in a bit rate NBR ranging from the 0 limit value to the maximum bit rate value $\mathrm{NBR_{max}}$. The result is shown in Fig. 7 as the dependence of the value of the ratio of maximum bit rate and the utilisation of the data link $\mathrm{NBR_{max}}/\mathrm{LU}$ on the value of the bit rate NBR.

When determining the adjustment factor κ, it is necessary to proceed from the ratio of the value of the maximum bit rate $\mathrm{NBR_{max}}$ determined on the basis of network traffic monitoring and the value of the data link utilisation parameter LU. In a specific example of the backhaul A, the maximum bit rate at peak time was $\mathrm{NBR_{max}} = 302{,}2\ \mathrm{Mb}/s$, which represents the value of the parameter $\mathrm{LU_{max}} \doteq 0{,}302$. The value of the data link utilisation LU is generally linearly dependent on the current bit rate value NBR. As stated in Fig. 7, the dependence of the ratio $\mathrm{NBR_{max}}/\mathrm{LU}$ on the bit rate NBR is of a power nature and is significantly influenced by the maximum bit rate value $\mathrm{NBR_{max}}$, which at the same time delimits from above the assessed set of bit rates NBR. The value of the maximum bit rate $\mathrm{NBR_{max}}$ determined from network traffic monitoring also reflects the current utilisation of the data link capacity during peak time (within the timeframe under consideration).

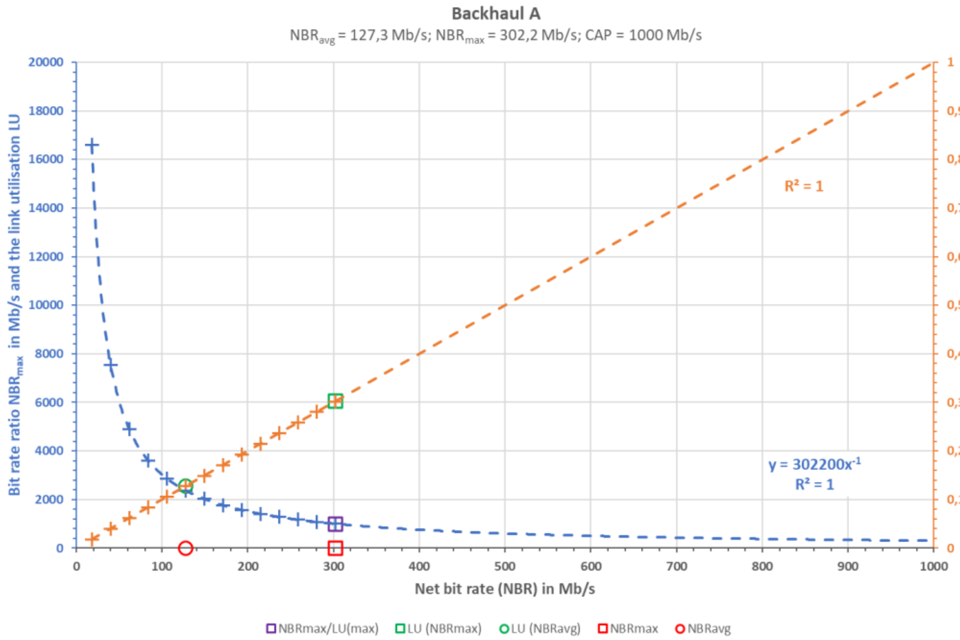

Fig. 7: The relationship between the value of the bit rate ratio $\mathrm{NBR_{max}}$ (Mb/s) and the link utilisation LU on the value of the bit rate NBR in the example of the backhaul A; the link capacity corresponds to $\mathrm{CAP} = 1000\,\mathrm{Mb/s}$; the figure shows the values of the backhaul A according to the monthly network traffic monitoring (Tab. 2)

The adjustment factor κ determines according to the relationship (33) the speed of theoretical increase of the original capacity value CAP to the adjusted capacity value CAP', because the initial relationship (21) always assumes the maximum capacity utilisation of the assessed data link. When determining the adjustment factor κ, the existing utilisation of the data link capacity shall be taken into account according to the results of network traffic monitoring which is delimited at peak time by the value of the maximum bit rate $\mathrm{NBR_{max}}$:

$$\kappa = \frac{\frac{\mathrm{NBR_{max}}}{\mathrm{LU}}}{\mathrm{NBR_{max}}} = \frac{\mathrm{NBR_{max}}}{\mathrm{LU}\cdot\mathrm{NBR_{max}}} = \frac{1}{\mathrm{LU}}. \tag{34}$$

The value of the adjustment factor κ is the inverted value of the data link utilisation LU. In general, as the bit rate NBR rises, the value of the LU parameter rises linearly, while the value of the adjustment factor κ decreases, see Fig. 8. The relationship (33) can be modified as:

$$\mathrm{CAP}' = \frac{\mathrm{CAP}}{\mathrm{LU}}. \tag{35}$$

In a situation where the bit rate NBR equals to the data link capacity CAP, the value $\mathrm{LU} = 1$ and thus the value $\kappa = 1$. Therefore, there is no theoretical increase in capacity, the data link utilisation is at its maximum and therefore, the situation mentioned in the relationship (32) occurs. If we put into the relationship (34) the values from the example of the backhaul A, which reached the peak time value of the maximum bit rate $\mathrm{NBR_{max}} = 302{,}2\,\mathrm{Mb/s}$, which represents the value of the parameter $\mathrm{LU_{max}} \doteq 0{,}302$ (see Fig. 8), we can determine the adjustment factor κ as:

$$\kappa = \frac{\frac{\mathrm{NBR_{max}}}{\mathrm{LU_{max}}}}{\mathrm{NBR_{max}}} = \frac{1}{\mathrm{LU_{max}}} = \frac{1}{0{,}302} \doteq 3{,}307. \tag{36}$$

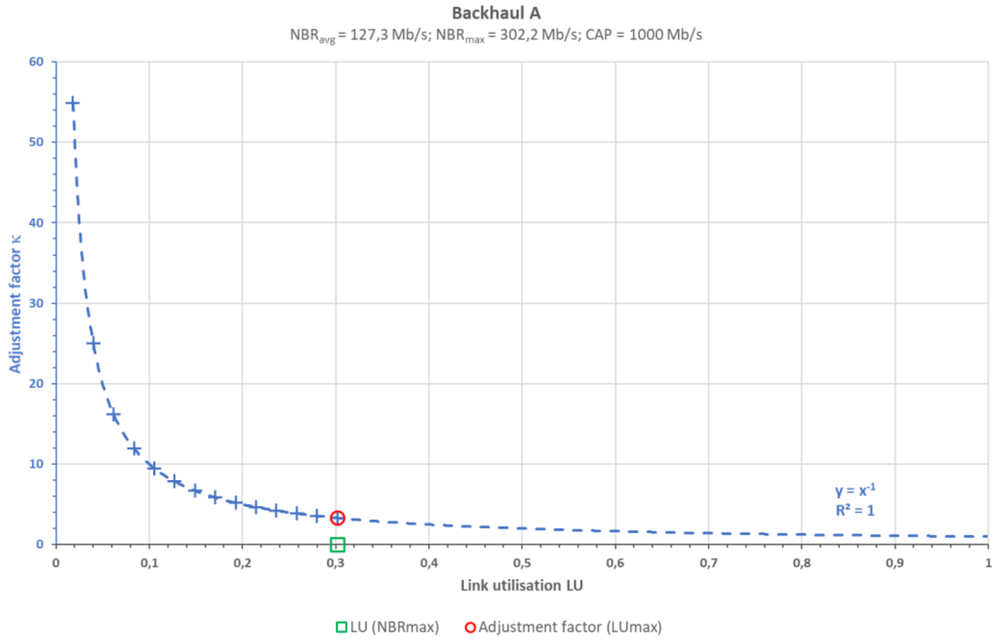

Fig. 8: The dependency of the adjustment factor κ on the data link utilisation LU in the example of backhaul A; the link capacity corresponds to $1000\ \mathrm{Mb/s}$

In order to take into account the value of the utilisation factor UF in the calculation of the value of the adjusted capacity CAP', the relationship (34) may be further adjusted, whereas given the method of determining the value of the utilisation factor UF based on network traffic monitoring (maximum bit rate $\mathrm{NBR_{max}}$ and average bit rate $\mathrm{NBR_{avg}}$ over the timeframe monitored), the value of the data link utilisation LU in the relationship (34) corresponds to the specific value of the maximum bit rate $\mathrm{NBR_{max}}$, so that:

$$\kappa = \frac{1}{\mathrm{LU_{max}}} = \frac{1}{\frac{\mathrm{NBR_{max}}}{\mathrm{CAP}}} = \frac{\mathrm{CAP}}{\mathrm{NBR_{max}}} \cdot \frac{\mathrm{NBR_{avg}}}{\mathrm{NBR_{avg}}} = \frac{\mathrm{CAP{\cdot}UF}}{\mathrm{NBR_{avg}}}. \quad (37)$$

The resulting value of the adjusted capacity CAP' can be determined by adding the expression of the adjustment factor κ according to the relationship (37) into the relationship (33):

$$\mathrm{CAP}' = \kappa \cdot \mathrm{CAP} = \frac{\mathrm{CAP{\cdot}UF}}{\mathrm{NBR_{avg}}} \cdot \mathrm{CAP} = \frac{\mathrm{CAP^2{\cdot}UF}}{\mathrm{NBR_{avg}}}. \quad (38)$$

The result of the relationship (37) and the relationship (38), taking into account the change in the value of the utilisation factor UF, is shown in Fig. 9. For the value of the utilisation factor UF of the backhaul A example (Tab. 2), the CAP' corresponds to:

$$\mathrm{CAP}' = \frac{\mathrm{CAP^2{\cdot}UF}}{\mathrm{NBR_{avg}}} = \frac{1000^2{\cdot}0{,}421}{127{,}3} \doteq 3307\ \mathrm{Mb/s}. \quad (39)$$

After inserting the relationship (38), the resulting value of the average number of network termination points NTP according to the Poisson process, derived from the relationship (21), can be written down in a general form as:

$$\mathrm{NTP} = \left\lfloor \frac{\mathrm{CAP}'{\cdot}\lambda}{\mathrm{AAS}} \right\rfloor = \left\lfloor \frac{\kappa{\cdot}\mathrm{CAP}{\cdot}\lambda}{\mathrm{AAS}} \right\rfloor = \left\lfloor \frac{\frac{\mathrm{CAP^2{\cdot}UF}}{\mathrm{NBR_{avg}}}{\cdot}\lambda}{\mathrm{AAS}} \right\rfloor = \left\lfloor \frac{\mathrm{UF{\cdot}CAP^2{\cdot}\lambda}}{\mathrm{AAS{\cdot}NBR_{avg}}} \right\rfloor\ [-;-,(\mathrm{Mb/s})^2,-,\mathrm{Mb/s},\mathrm{Mb/s}]. \quad (40)$$

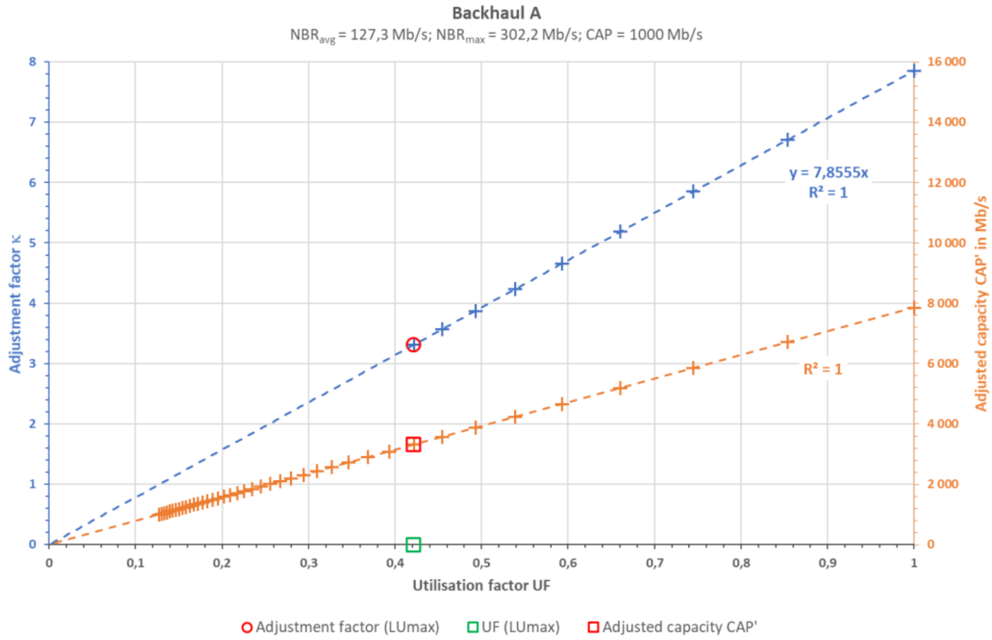

Fig. 9: The dependency of the adjusted capacity CAP' and adjustment factor κ on the value of the utilisation factor UF in the case of the example of backhaul A

It is clear that the relationship (40) for the calculation of the average number of NTP according to the Poisson process depends not only on knowledge of the utilisation factor UF, but also on the knowledge of the value of the average bit rate $\mathrm{NBR}_{\mathrm{avg}}$ (see mathematical expression of the regression equation for the adjustment factor κ in Fig. 9). Furthermore, the relationship (40) shows, as in the case of the initial general relationship (21), that it is necessary to recalculate the value of capacity CAP and the value of the average bit rate $\mathrm{NBR}_{\mathrm{avg}}$ from the physical layer according to the ISO/OSI reference model to the transport layer. For comparison, Fig. 10 shows the dependence of the adjustment factor κ (correction) on the utilisation factor UF of given examples of backhauls according to Tab. 2 (A, B, C and D).

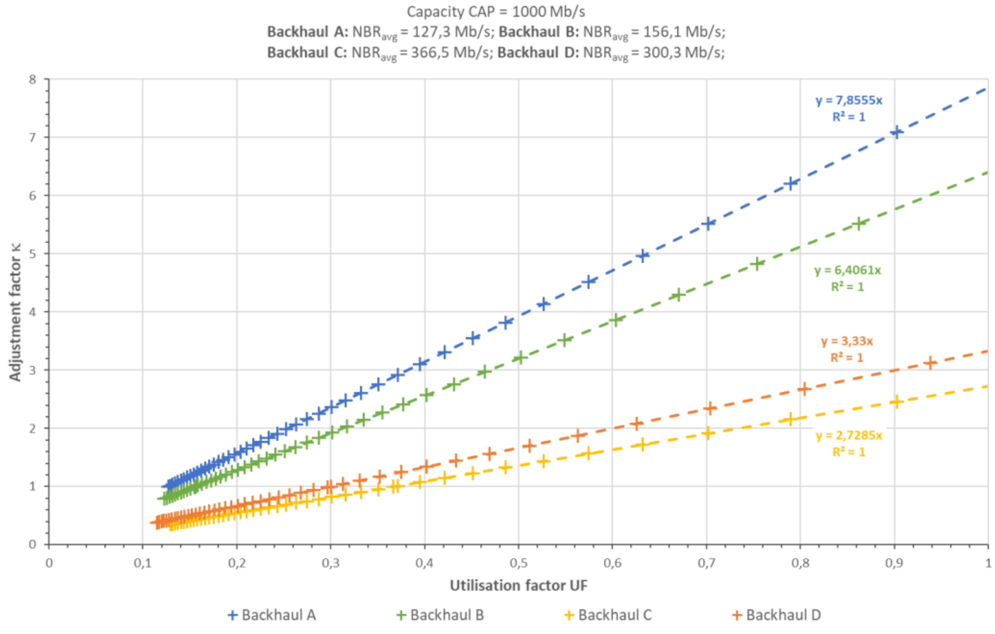

Fig. 10: The dependency of the adjustment factor (correction) κ on the value of the utilisation factor UF in the case of the examples of backhauls (A, B, C and D) according to Tab. 2, the decisive parameters are the values of the rate $\mathrm{NBR}_{\mathrm{avg}}$ and the link capacity $\mathrm{CAP} = 1000$ Mb/s

If we return to the example of calculating the average number of NTP in the case of GPON access technology, see the relationship (22), assuming the monthly parameters of distribution network as in the case of backhaul A, see Tab. 2, respectively $\mathrm{NBR_{avg}} = 127{,}3\ \mathrm{Mb/s}$ and $\mathrm{UF} = 0{,}421$, while maintaining the same capacity $\mathrm{CAP} = 1000\ \mathrm{Mb/s}$, the average number of NTP will change for $\mathrm{AAS} = 60\ \mathrm{Mb/s}$ from 176 to 582, see calculation (41). It is necessary not to forget to recalculate the value of average bit rate $\mathrm{NBR_{avg}}$ from the physical layer of the ISO/OSI model to the transport layer.

$$\mathrm{NTP_{A\_GPON}} = \left\lfloor \frac{\mathrm{UF \cdot CAP^2 \cdot \lambda}}{\mathrm{AAS \cdot NBR_{avg}}} \right\rfloor = \left\lfloor \frac{0{,}421 \cdot 949{,}285^2 \cdot 11{,}140}{60 \cdot 120{,}844} \right\rfloor = 582\ [-;-,(\mathrm{Mb/s})^2,-,\mathrm{Mb/s},\mathrm{Mb/s}]. \quad (41)$$

In the case of the monthly parameters of distribution network for the example of GPON technology as shown by the backhaul D, see Tab. 2, respectively $\mathrm{NBR_{avg}} = 300{,}3\ \mathrm{Mb/s}$ and $\mathrm{UF} = 0{,}375$, while maintaining the same capacity $\mathrm{CAP} = 1000\ \mathrm{Mb/s}$, the average number of NTP will change from 176 to 220, respectively:

$$\mathrm{NTP_{D\_GPON}} = \left\lfloor \frac{\mathrm{UF \cdot CAP^2 \cdot \lambda}}{\mathrm{AAS \cdot NBR_{avg}}} \right\rfloor = \left\lfloor \frac{0{,}375 \cdot 949{,}285^2 \cdot 11{,}140}{60 \cdot 285{,}070} \right\rfloor = 220\ [-;-,(\mathrm{Mb/s})^2,-,\mathrm{Mb/s},\mathrm{Mb/s}]. \quad (42)$$

The conversion characteristic of the example of GPON technology at the distribution network capacity corresponding to $1000\ \mathrm{Mb/s}$ (L 1), the parameters of the utilisation factor $\mathrm{UF} = 0{,}421$ and average bit rate $\mathrm{NBR_{avg}} = 127{,}3\ \mathrm{Mb/s}$ taken from the example of the backhaul A, see Tab. 2, is given in Fig. 11. In this representation of the conversion characteristic, it can be noticed that in the case of $\mathrm{NTP} = 64$, a typical basic dividing ratio $1{:}64$ of passive fibre distribution network, the AAS value has changed from $94{,}931\ \mathrm{Mb/s}$ to $161{,}83\ \mathrm{Mb/s}$ (values derived from the waveform of the regression characteristic; red points) due to the utilisation factor UF.

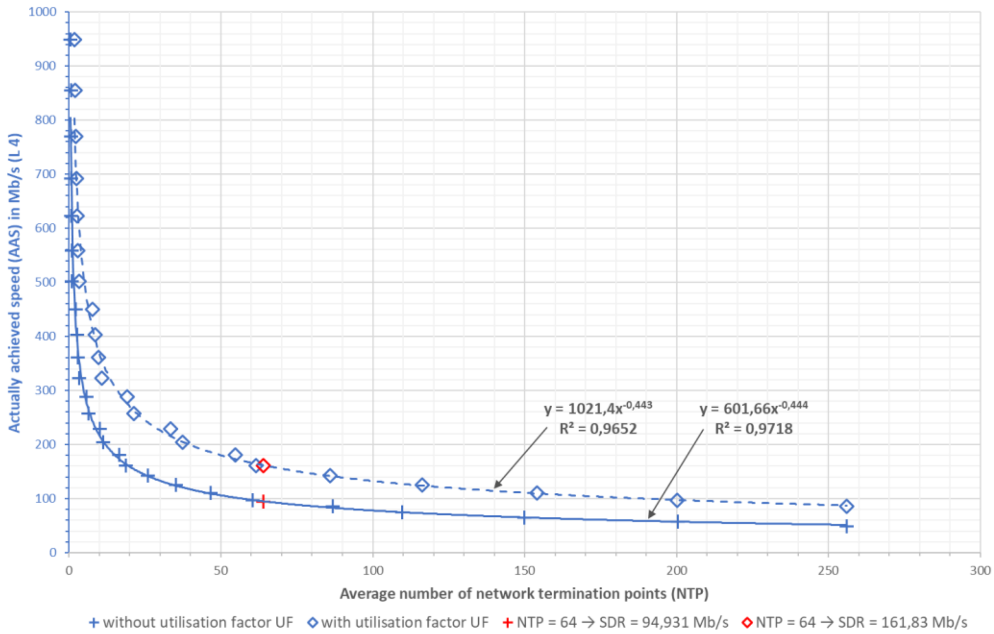

Fig. 11: The dependency of AAS value on average number of NTP, an example of GPON technology with backhaul capacity $\mathrm{CAP} = 1000\ \mathrm{Mb/s}$ (L 1); parameters $\mathrm{UF} = 0{,}421$ and $\mathrm{NBR_{avg}} = 127{,}3\ \mathrm{Mb/s}$ taken from backhaul A are taken into account, see Tab. 2

In the second case of 5G network technology operating in the $3{,}5\ \mathrm{GHz}$ band with bandwidth $\mathrm{B} = 100\ \mathrm{MHz}$, where the capacity of the backhaul corresponds to the capacity determined on the basis of dimensioning cell capacity in the downlink direction (without an assessment of the impact of MIMO), and considering the physical principles of 5G network technology behaviour, i.e., the value $892{,}5\ \mathrm{Mb/s}$ (L 3), see the relationship (23), the calculation of the average number of NTP will change on the basis of the relationship (43), assuming the same parameters as in the case of the example of backhaul A, see Tab. 2, respectively $\mathrm{NBR_{avg}} = 127{,}3\ \mathrm{Mb}/s$ and $\mathrm{UF} = 0{,}421$. The resulting change in the calculation will mean an increase in the average number of NTP from 151 to 463, see calculation (43). Again, it is necessary not to forget to recalculate the value of the average bit rate $\mathrm{NBR_{avg}}$ from the physical layer of the ISO/OSI model to the transport layer.

$$\mathrm{NTP_{A\_5G}} = \left\lfloor \frac{\mathrm{UF \cdot CAP^2 \cdot \lambda}}{\mathrm{AAS \cdot NBR_{avg}}} \right\rfloor = \left\lfloor \frac{0{,}421 \cdot 880{,}439^2 \cdot 10{,}305}{60 \cdot 120{,}844} \right\rfloor = 463\ [-; -, (\mathrm{Mb/s})^2, -, \mathrm{Mb/s}, \mathrm{Mb/s}]. \quad (43)$$

In the case of the same distribution network parameters for the example of 5G network technology as shown by backhaul D, see Tab. 2, respectively, $\mathrm{NBR_{avg}} = 300{,}3\ \mathrm{Mb}/s$ and $\mathrm{UF} = 0{,}375$, while maintaining the same capacity $\mathrm{CAP} = 1000\ \mathrm{Mb/s}$, the average number of NTP will change from 151 to 175, respectively:

$$\mathrm{NTP_{D\_5G}} = \left\lfloor \frac{\mathrm{UF \cdot CAP^2 \cdot \lambda}}{\mathrm{AAS \cdot NBR_{avg}}} \right\rfloor = \left\lfloor \frac{0{,}375 \cdot 880{,}439^2 \cdot 10{,}305}{60 \cdot 285{,}070} \right\rfloor = 175\ [-; -, (\mathrm{Mb/s})^2, -, \mathrm{Mb/s}, \mathrm{Mb/s}]. \quad (44)$$

The conversion characteristic of the example of 5G network technology operating in the $3{,}5\ \mathrm{GHz}$ with bandwidth $\mathrm{B} = 100\ \mathrm{MHz}$, at a capacity corresponding to capacity determined on the basis of dimensioning cell capacity in the downlink direction and service utilisation parameters $\mathrm{UF} = 0{,}421$ and average bit rate $\mathrm{NBR_{avg}} = 127{,}3\ \mathrm{Mb}/s$ taken from the example of backhaul A, see Tab. 2, is given in Fig. 12. Should this FWA technology in that configuration provide a value $\mathrm{NAS} = 100\ \mathrm{Mb}/s$ (L 4) for its terminal equipment, it would be able to do so with probability 90% up to the average number of $\mathrm{NTP} = 149$ against the original number of $\mathrm{NTP} = 47$ (according to the regression characteristic).

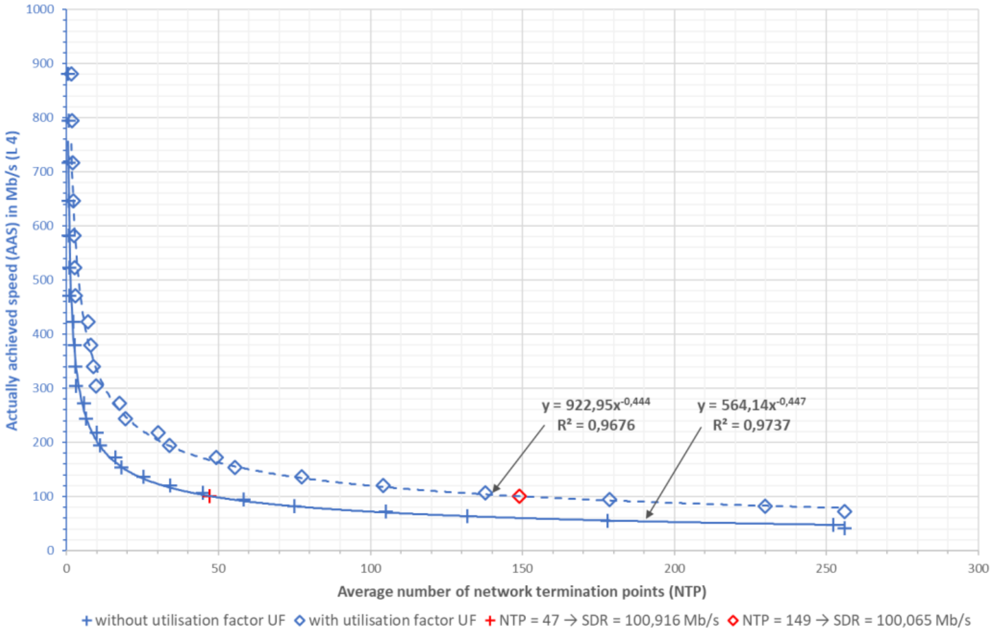

Fig. 12: The dependency of AAS value on average number of NTP, example of FWA technology (5G in the $3{,}5\ \mathrm{GHz}$ band) with available backhaul capacity ≥ $892{,}5\ \mathrm{Mb}/s$ (L 3); parameters $\mathrm{UF} = 0{,}421$ and $\mathrm{NBR_{avg}} = 127{,}3\ \mathrm{Mb}/s$ taken from backhaul A are taken into account, see Tab. 2; without an assessment of the impact of MIMO

3. Performance of internet access service (AAS)

When assessing the impact of network capacity on the resulting performance of internet access services represented by AAS at the NTP, it shall be proceed from the actual knowledge of the network capacity, as in assessing the impact on the average number of NTP. In this case, the procedure is the same as in subchapter *Number of Network Terminal Points (NTP)*, therefore the initial process involving identification of network capacity value and its conversion to transport layer using TCP protocol will be skipped. If the network capacity is known, it is necessary to specify further the number of NTP. Access network corresponding to the ITU-T G.984 technology, or GPON, with the number of ONT terminal units corresponding to the number of $\mathrm{NTP} = 64$, i.e. the basic dividing ratio $1{:}\,64$ (aggregation) of the passive fibre distribution network, can also be an example. Following the above mentioned conversion between the different layers of the ISO/OSI model, it is assumed that the central unit OLT of GPON technology is connected to the backhaul via the SFP module for a fiber-optic cable of IEEE 802.3z (1000BASE-SX) standard with a capacity corresponding to bit speed $\mathrm{NBR} = 1000\ \mathrm{Mb}/s$ (L 1), i.e. speed $\mathrm{TCP\ TR} = 949{,}285\ \mathrm{Mb}/s$ (L 4).

Furthermore, it is necessary to determine the percentage of probability value $\mathrm{P(k \leq N)}$, in which a certain number of data flows generated by a specified number of ONTs (NTP) will come to the queueing system (GPON). It is generally recommended that the probability value should not be lower than 90%. The probability limit value $\mathrm{P(k \leq N)} = 90\%$ is selected for the purposes of this case. With regard to the relationship (17), describing the calculation of the input parameter N of the Poisson process, it is apparent that without knowledge of the AAS value, the input parameter cannot be determined on the basis of the relationship (17) and has to be mathematically derived. For mathematical derivation of an unknown AAS value, the relationship (21) shall be used:

$$\mathrm{NTP} = \left\lfloor \frac{\mathrm{CAP} \cdot \lambda}{\mathrm{AAS}} \right\rfloor \Rightarrow \mathrm{AAS} = \frac{\mathrm{CAP} \cdot \lambda}{\mathrm{NTP}}\ [\mathrm{Mb/s;\ Mb/s}, -, -]. \tag{45}$$

If the mathematical derivation of the AAS value is inserted back in the relationship (17), the value of the input parameter N can be written down in the form:

$$\mathrm{AAS_{min}} = \mathrm{AAS}\ \Rightarrow \mathrm{N} = \left\lfloor \frac{\mathrm{CAP}}{\mathrm{AAS}} \right\rfloor = \left\lfloor \frac{\mathrm{CAP}}{\frac{\mathrm{CAP} \cdot \lambda}{\mathrm{NTP}}} \right\rfloor = \left\lfloor \frac{\mathrm{CAP} \cdot \mathrm{NTP}}{\mathrm{CAP} \cdot \lambda} \right\rfloor = \left\lfloor \frac{\mathrm{NTP}}{\lambda} \right\rfloor = \left\lfloor \frac{64}{\lambda} \right\rfloor\ [-; -] \tag{46}$$

where it can be said that the input parameter N equals to the ratio of the number of NTP and mean input intensity λ of the Poisson probability distribution, which expresses the average number of data flows that entered the queueing system per unit of time t. The mean input intensity parameter λ can be determined for time $\mathrm{t} = 1$ using the following relation (14):

$$\mathrm{P(k \leq N)} = \sum_{\mathrm{k}=0}^{\mathrm{N}} \frac{(\lambda)^{\mathrm{k}}}{\mathrm{k}!} \mathrm{e}^{-\lambda} \geq 90\% \Rightarrow \mathrm{P}\left(\mathrm{k} \leq \left\lfloor \frac{64}{\lambda} \right\rfloor\right) = \sum_{\mathrm{k}=0}^{\left\lfloor \frac{64}{\lambda} \right\rfloor} \frac{(\lambda)^{\mathrm{k}}}{\mathrm{k}!} \mathrm{e}^{-\lambda} \geq 0{,}9 \Rightarrow$$
$$\Rightarrow \lambda \doteq 6{,}485\ [-]. \tag{47}$$

The value of the mean input intensity λ can then be inserted back in the relationship (45) to determine the AAS value. In the example of GPON technology with a number of $\mathrm{NTP} = 64$ and based on the theory of a queueing system according to Poisson distribution can be stated that AAS value will reach the average value $96{,}189\ \mathrm{Mb/s}$: with the probability value $\mathrm{P(k \leq N)} = 90\%$:

$$\mathrm{AAS} = \frac{\mathrm{CAP} \cdot \lambda}{\mathrm{NTP}} = \frac{949{,}285 \cdot 6{,}485}{64} \doteq 96{,}189\ [\mathrm{Mb/s;\ Mb/s}, -, -]. \tag{48}$$

Where it is necessary to take into account the influence of the utilisation factor UF of an existing data link, a backhaul or distribution network (generally a *bottleneck*) and its values characterising end-users behaviour at a given location and type of data traffic, it is necessary to make an adequate adjustment of the initial relationship (45) according to the relationship (40). For the example of GPON technology with the number of network terminal points $\mathrm{NTP} = 64$, end-user behaviour parameters will be taken from a specific example of backhaul A, where the value of the average bit rate $\mathrm{NBR_{avg}} = 127{,}3\ \mathrm{Mb}/s$ and the value of the utilisation factor $\mathrm{UF} = 0{,}421$ is based on Tab. 2. The initial relationship (45) can therefore be adjusted to:

$$\mathrm{NTP} = \left\lfloor \frac{\mathrm{CAP}' \cdot \lambda}{\mathrm{AAS}} \right\rfloor \Longrightarrow \mathrm{AAS} = \frac{\mathrm{CAP}' \cdot \lambda}{\mathrm{NTP}} = \frac{\kappa \cdot \mathrm{CAP} \cdot \lambda}{\mathrm{NTP}} = \frac{\frac{\mathrm{CAP}}{\mathrm{NBR_{avg}}} \cdot \mathrm{UF} \cdot \mathrm{CAP} \cdot \lambda}{\mathrm{NTP}} =$$

$$= \frac{\mathrm{UF} \cdot \mathrm{CAP}^2 \cdot \lambda}{\mathrm{NTP} \cdot \mathrm{NBR_{avg}}} \ [\mathrm{Mb/s}; -, (\mathrm{Mb/s})^2, -, -, \mathrm{Mb/s}]. \tag{49}$$

Following the substitution of the relationship (49) into the relationship (17), the value of the input parameter N can be expressed in the specific case of GPON technology as:

$$\mathrm{AAS_{min}} = \mathrm{AAS} \ \Longrightarrow \mathrm{N} = \left\lfloor \frac{\mathrm{CAP}}{\mathrm{AAS}} \right\rfloor = \left\lfloor \frac{\mathrm{CAP}}{\frac{\mathrm{UF} \cdot \mathrm{CAP}^2 \cdot \lambda}{\mathrm{NTP} \cdot \mathrm{NBR_{avg}}}} \right\rfloor = \left\lfloor \frac{\mathrm{NTP} \cdot \mathrm{NBR_{avg}}}{\mathrm{UF} \cdot \mathrm{CAP} \cdot \lambda} \right\rfloor = \left\lfloor \frac{\mathrm{NBR_{avg}}}{\mathrm{UF} \cdot \mathrm{CAP}} \cdot \frac{\mathrm{NTP}}{\lambda} \right\rfloor =$$

$$= \left\lfloor \frac{127{,}3}{0{,}421 \cdot 1000} \cdot \frac{64}{\lambda} \right\rfloor = \left\lfloor 0{,}302 \cdot \frac{64}{\lambda} \right\rfloor \ [-; -, \mathrm{Mb/s}, -, \mathrm{Mb/s}, -]. \tag{50}$$

where it can be said that, as in the case of the relationship (46), the input parameter N equals to the ratio of the number of NTP and the mean input intensity λ extended by a constant, the value of which corresponds practically to the ratio of the maximum bit rate value $\mathrm{NBR_{max}}$ and the given data link capacity CAP, i.e. the value of the maximum data link utilisation $\mathrm{LU_{max}}$, see relationship (29). The correctness of the mathematical simplification of the resulting constant value from the relationship (50) to the value of maximum data link utilisation $\mathrm{LU_{max}}$ can be derived as follows:

$$\frac{\mathrm{NBR_{avg}}}{\mathrm{UF} \cdot \mathrm{CAP}} = \frac{\mathrm{NBR_{avg}}}{\frac{\mathrm{NBR_{avg}}}{\mathrm{NBR_{max}}} \cdot \mathrm{CAP}} = \frac{\mathrm{NBR_{avg}} \cdot \mathrm{NBR_{max}}}{\mathrm{NBR_{avg}} \cdot \mathrm{CAP}} = \frac{\mathrm{NBR_{max}}}{\mathrm{CAP}} = \mathrm{LU_{max}} \ [-; \mathrm{Mb/s}, \mathrm{Mb/s}] \tag{51}$$

thus, for the example mentioned above, the verification of the validity of the relationship (51) can be calculated:

$$\mathrm{LU_{max}} = \frac{\mathrm{NBR_{max}}}{\mathrm{CAP}} = \frac{302{,}2}{1000} \doteq 0{,}302 \ [-; \mathrm{Mb/s}, \mathrm{Mb/s}]. \tag{52}$$

Obviously, the value of the result of the simplified relationship (52) is the same as the value of the constant given on the basis of the initial relationship calculation (50), i.e. $0{,}302$. The mean input intensity parameter λ can then be determined for the time $\mathrm{t} = 1$ using the relationship (14):

$$\mathrm{P}(\mathrm{k} \leq \mathrm{N}) = \sum_{\mathrm{k}=0}^{\mathrm{N}} \frac{(\lambda)^{\mathrm{k}}}{\mathrm{k}!} \mathrm{e}^{-\lambda} \geq 90\% \Rightarrow \mathrm{P}\left(\mathrm{k} \leq \left\lfloor \mathrm{LU_{max}} \cdot \frac{\mathrm{NTP}}{\lambda} \right\rfloor\right) = \sum_{\mathrm{k}=0}^{\left\lfloor \mathrm{LU_{max}} \cdot \frac{\mathrm{NTP}}{\lambda} \right\rfloor} \frac{(\lambda)^{\mathrm{k}}}{\mathrm{k}!} \mathrm{e}^{-\lambda} \geq 0{,}9 =$$

$$\sum_{\mathrm{k}=0}^{\left\lfloor 0{,}302 \cdot \frac{64}{\lambda} \right\rfloor} \frac{(\lambda)^{\mathrm{k}}}{\mathrm{k}!} \mathrm{e}^{-\lambda} \geq 0{,}9 \Longrightarrow \ \lambda \doteq 3{,}350 \ [-; -, -, -]. \tag{53}$$

The value of the mean input intensity λ can then be inserted back in the relationship (50) to determine the average AAS value. In the example of GPON technology with the number $\mathrm{NTP} = 64$ and parameters of the end-users behaviour taken from a particular example of the backhaul A, see Tab. 2, i.e. the value of the average bit rate $\mathrm{NBR_{avg}} = 127{,}3 \ \mathrm{Mb/s}$ and the value of the utilisation factor $\mathrm{UF} = 0{,}421$, and based on the theory of a queueing system according to Poisson distribution can be stated that AAS will reach the average value with the probability value $\mathrm{P}(\mathrm{k} \leq \mathrm{N}) = 90\%$:

$$\mathrm{AAS} = \frac{\mathrm{UF} \cdot \mathrm{CAP}^2 \cdot \lambda}{\mathrm{NTP} \cdot \mathrm{NBR_{avg}}} = \frac{0{,}421 \cdot 949{,}285^2 \cdot 3{,}350}{64 \cdot 120{,}844} \doteq 164{,}329 \ [\mathrm{Mb/s}; \ \mathrm{Mb/s}, -, -]. \tag{54}$$

From a global perspective, the performance of an internet access service, represented by the average AAS value, can also be assessed in terms of the resulting impact of the combination of the capacity value and the corresponding aggregation ratio, in the form of a variation of the service performance. Again, the above example of GPON technology will be used for explanation. If we mark the lowest possible average value $\mathrm{AAS_{min}}$, which is reached at NTP in time when all end-users of the internet access service use the service as much as possible, as a specific value of the normally available speed $\mathrm{NAS} = 100\ \mathrm{Mb/s}$, it can be determined from the relationship (17):

$$\mathrm{NAS} = \mathrm{AAS_{min}} = 100\ \mathrm{Mb/s} \Rightarrow \mathrm{N} = \left\lfloor \frac{\mathrm{CAP}}{\mathrm{NAS}} \right\rfloor = \left\lfloor \frac{949{,}285}{100} \right\rfloor = 9\ [;\mathrm{Mb/s}, \mathrm{Mb/s}]. \tag{55}$$

If we put the number N into the relationship (14) with probability value $\mathrm{P}(\mathrm{k} \leq \mathrm{N}) = 90\%$, we get a mean input intensity value λ corresponding to the average number of NTP generating data stream over time $\mathrm{t} = 1$, which in other words, will utilise the internet access service with 90% probability:

$$\mathrm{P}(\mathrm{k} \leq \mathrm{N}) = \sum_{\mathrm{k}=0}^{\mathrm{N}} \frac{(\lambda)^{\mathrm{k}}}{\mathrm{k}!} \mathrm{e}^{-\lambda} \geq 90\% \Rightarrow \mathrm{P}(\mathrm{k} \leq 9) = \sum_{\mathrm{k}=0}^{9} \frac{(\lambda)^{\mathrm{k}}}{\mathrm{k}!} \mathrm{e}^{-\lambda} \geq 0{,}9 \rightarrow$$

$$\Rightarrow \lambda \doteq 6{,}215\ [-]. \tag{56}$$

Knowing the mean input intensity value λ, the performance of the internet access service $\mathrm{AAS_{max}}$ shall be determined according to the relationship (19) as:

$$\mathrm{AAS_{max}} = \frac{\mathrm{CAP}}{\lambda} = \frac{949{,}285}{6{,}215} \doteq 152{,}741\ [\mathrm{Mb/s}; \mathrm{Mb/s}, -]. \tag{57}$$

The variation of performance of the internet access service, i.e. the average AAS value reached at NTP, shall be determined as the difference between the maximum value $\mathrm{AAS_{max}}$ and the minimum value $\mathrm{AAS_{min}}$:

$$\sigma_{\mathrm{AAS}} = \mathrm{AAS_{max}} - \mathrm{AAS_{min}} = 152{,}741 - 100 = 52{,}741\ [\mathrm{Mb/s}; \mathrm{Mb/s}, \mathrm{Mb/s}]. \tag{58}$$

In order to better interpret the variation value of the performance of the internet access service, a percentage ratio of the variation value σ_{AAS} and value $\mathrm{AAS_{max}}$ is determined, which can be described as a decrease in the average value of AAS ($\mathrm{P_{AAS}}$):

$$\mathrm{P_{AAS}} = \frac{\sigma_{\mathrm{AAS}}}{\mathrm{AAS_{max}}} \cdot 100 = \frac{52{,}741}{152{,}741} \cdot 100 \doteq 34{,}53\ [\%; \mathrm{Mb/s}, \mathrm{Mb/s}]. \tag{59}$$

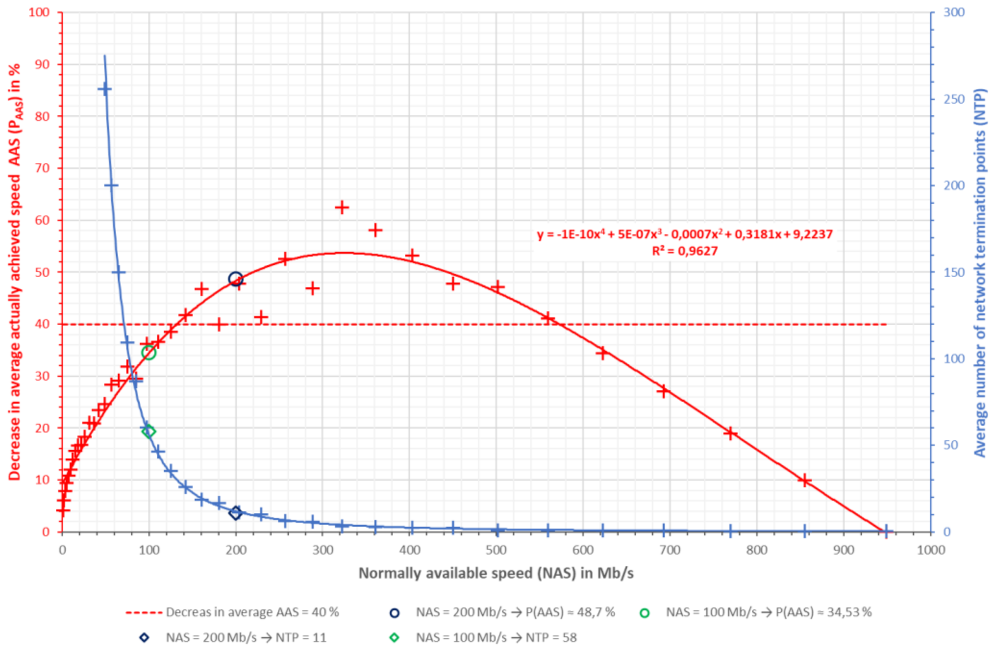

Fig. 13: The dependency of the decrease in average speed AAS ($\mathrm{P_{AAS}}$) and the average number of network termination points NTP on normally available speed (NAS), an example of GPON access technology with network capacity $1000\ \mathrm{Mb/s}$ (L 1); the network utilisation factor UF and average bit rate $\mathrm{NBR_{avg}}$ not taken into account

If applies that the value $\mathrm{NAS} = \mathrm{AAS_{min}}$, see the relationship (17), the dependence of the decline in the performance of the internet access service $\mathrm{P_{AAS}}$ and the average number of NTP on the

NAS value can be displayed for the above mentioned example of GPON access technology, as shown in Fig. 13. From the result, for example, it can be derived that with the backhaul capacity $\mathrm{CAP} = 1000\ \mathrm{Mb/s}$, the given GPON technology can provide $\mathrm{NAS} = 200\ \mathrm{Mb/s}$ for the average number of $\mathrm{NTP} = 11$, but the decrease of the average value of AAS will reach $\mathrm{P_{AAS}} = 48{,}7\ \%$. According to Fig. 13, the value $\mathrm{NAS} = 100\ \mathrm{Mb/s}$ can be ensured with an average number of $\mathrm{NTP} = 58$, with a decrease in average value of AAS will reach $\mathrm{P_{AAS}} = 34{,}53\%$, see the relationship (59). Again, in this case, all NTP utilise internet access services at the maximum possible transmission speed at a given time $\mathrm{t} = 1$, see the relationship (14).

In the case where it is required to take into account the influence of the utilisation factor UF of an existing data link, or a backhaul or distribution network (generally a *bottleneck*) and its values characterising the end-users behaviour at a given location and type of data traffic, it is necessary to make an adequate adjustment of the initial relationship (55) according to the relationship (40). For the example of GPON technology, end-user behaviour parameters will be taken from a specific example of backhaul A, where the value of the average bit rate $\mathrm{NBR_{avg}} = 127{,}3\ \mathrm{Mb/s}$ and the value of the utilisation factor $\mathrm{UF} = 0{,}421$ are based on Tab. 2. The value of the average bit rate $\mathrm{NBR_{avg}}$ must be recalculated to the transport layer. The initial relationship (55) can therefore be adjusted as follows:

$$\mathrm{AAS_{min}} = \mathrm{NAS} = 100\ \mathrm{Mb/s} \Rightarrow \mathrm{N} = \left\lfloor \frac{\mathrm{CAP}'}{\mathrm{NAS}} \right\rfloor = \left\lfloor \frac{\kappa \cdot \mathrm{CAP}}{\mathrm{NAS}} \right\rfloor = \left\lfloor \frac{\frac{\mathrm{CAP}}{\mathrm{NBR_{avg}}} \cdot \mathrm{UF} \cdot \mathrm{CAP}}{\mathrm{NAS}} \right\rfloor = \left\lfloor \frac{\mathrm{UF} \cdot \mathrm{CAP}^2}{\mathrm{NAS} \cdot \mathrm{NBR_{avg}}} \right\rfloor =$$

$$= \left\lfloor \frac{0{,}421 \cdot 949{,}285^2}{100 \cdot 120{,}844} \right\rfloor = 31\ [-; -, (\mathrm{Mb/s})^2, \mathrm{Mb/s}, \mathrm{Mb/s}]. \tag{60}$$

In the same way, it is necessary to determine, by relationship (56), the mean input intensity λ corresponding to the average number of NTP generating data flow over time $\mathrm{t} = 1$, which will utilise the internet access service with the probability value $\mathrm{P}(\mathrm{k} \leq \mathrm{N}) = 90\%$:

$$\mathrm{P}(\mathrm{k} \leq \mathrm{N}) = \sum_{\mathrm{k}=0}^{\mathrm{N}} \frac{(\lambda)^{\mathrm{k}}}{\mathrm{k}!} \mathrm{e}^{-\lambda} \geq 90\% \Rightarrow \mathrm{P}(\mathrm{k} \leq 31) = \sum_{\mathrm{k}=0}^{31} \frac{(\lambda)^{\mathrm{k}}}{\mathrm{k}!} \mathrm{e}^{-\lambda} \geq 0{,}9 \Rightarrow$$

$$\Rightarrow \lambda \doteq 25{,}050\ [-]. \tag{61}$$

In the same way as in the case of the relationship (60), it is necessary to adjust the relationship (57) using knowledge of the mean input intensity λ according to the above-mentioned relationship (61). The performance of the internet access service $\mathrm{AAS_{max}}$ can therefore be defined as:

$$\mathrm{AAS_{max}} = \frac{\mathrm{CAP}'}{\lambda} = \frac{\kappa \cdot \mathrm{CAP}}{\lambda} = \frac{\frac{\mathrm{CAP}}{\mathrm{NBR_{avg}}} \cdot \mathrm{UF} \cdot \mathrm{CAP}}{\lambda} = \frac{\mathrm{UF} \cdot \mathrm{CAP}^2}{\lambda \cdot \mathrm{NBR_{avg}}} =$$

$$= \frac{0{,}421 \cdot 949{,}285^2}{25{,}050 \cdot 120{,}844} \doteq 125{,}326\ [\mathrm{Mb/s}; -, (\mathrm{Mb/s})^2, -, \mathrm{Mb/s}]. \tag{62}$$

The variation of the performance of the internet access service, i.e. the average value AAS reached at NTP, shall again be determined as the difference between the maximum value $\mathrm{AAS_{max}}$ and the minimum value $\mathrm{AAS_{min}}$:

$$\sigma_{\mathrm{AAS}} = \mathrm{AAS_{max}} - \mathrm{AAS_{min}} = 125{,}326 - 100{,}000 = 25{,}326\ [\mathrm{Mb/s}; \mathrm{Mb/s}, \mathrm{Mb/s}]. \tag{62}$$

The resulting decrease in the average value AAS ($\mathrm{P_{AAS}}$) is determined on the basis of the relationship (59):

$$\mathrm{P_{AAS}} = \frac{\sigma_{\mathrm{AAS}}}{\mathrm{AAS_{max}}} \cdot 100 = \frac{25{,}326}{125{,}326} \cdot 100 \doteq 20{,}21\ [\%; \mathrm{Mb/s}, \mathrm{Mb/s}]. \tag{63}$$

If the value $\mathrm{NAS} = \mathrm{AAS_{min}}$, see relationship (17), the dependence of the variation of the performance of internet access service $\mathrm{P_{AAS}}$ and average number of NTP on the NAS value can be shown for the above example of GPON access, as shown in Fig. 14. Again, it can be derived from the result (blue intermittent waveform) that, with the distribution network capacity $\mathrm{CAP} = 1000\ \mathrm{Mb/s}$, the given GPON technology can provide $\mathrm{NAS} = 200\ \mathrm{Mb/s}$ at the average number of $\mathrm{NTP} = 38,$ and the decrease in the average value of AAS will reach only $\mathrm{P_{AAS}} = 29{,}00\ \%$. According to Fig. 14, the value $\mathrm{NAS} = 100\ \mathrm{Mb/s}$ can be ensured with an average number of $\mathrm{NTP} > 90$, namely $\mathrm{NTP} = 195$ (determined by calculation), and the decrease in the average value of AAS will reach $\mathrm{P_{AAS}} = 20{,}21\ \%$.

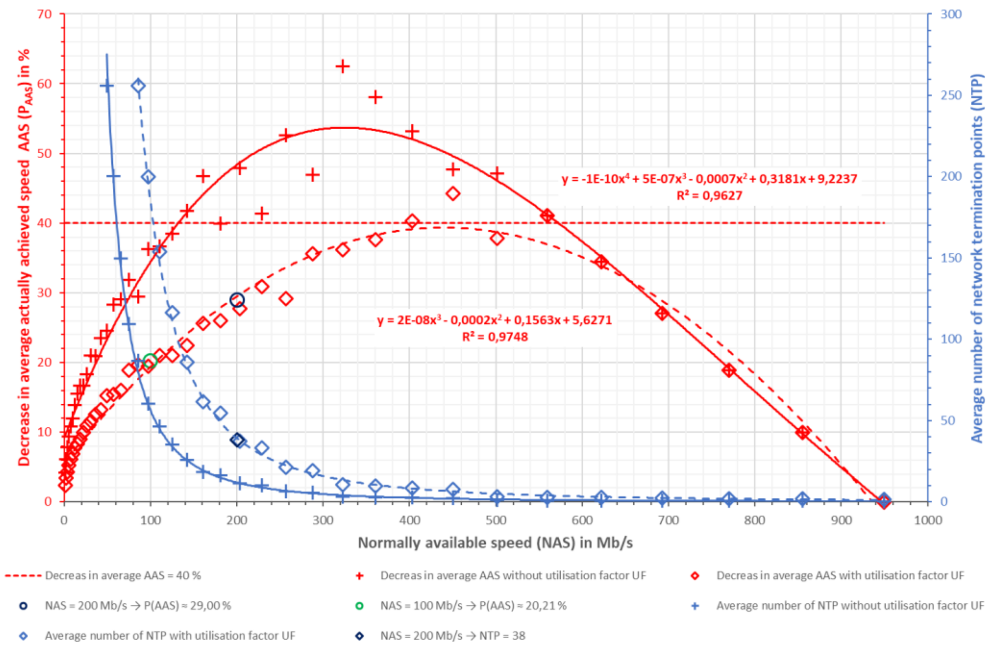

Fig. 14: The dependency of the decrease in average speed AAS ($\mathrm{P_{AAS}}$) and the average number of network termination points NTP on normally available speed (NAS), an example of GPON access technology with network capacity $\mathrm{CAP} = 1000\ \mathrm{Mb/s}$ (L 1); the utilisation factor $\mathrm{UF} = 0{,}421$ and average bitrate $\mathrm{NBR_{avg}} = 127{,}3\ \mathrm{Mb/s}$ taken from the backhaul A are taken into account, see Tab. 2

The conclusions of the chapter on the impact of the network utilisation factor by end-users on the performance of the internet access service can be explained by a simple example. There is a backhaul (distribution) network with capacity $\mathrm{CAP} = 1000\ \mathrm{Mb/s}$ (L 1), i.e. after conversion between the layers of the ISO/OSI model $949{,}285\ \mathrm{Mb/s}$ (L 4), and the number of network termination points $\mathrm{NTP} = 64$ (number of end-users, i.e. the number of connections). Should at one point, all 64 end-users (with the same device) use the internet access service in the maximum possible way, it would reach approximate value $\mathrm{AAS} \doteq 14{,}833\ \mathrm{Mb/s}$. Poisson process regulates the number of end-users (NTP) that will utilise the internet access service at the same time with the probability value $\mathrm{P(k \leq N)}$, but again, in the maximum possible way (maximum value of the transmission speed), and the limiting factor is the value of the backhaul (distribution) network capacity CAP. For the chosen probability value $\mathrm{P(k \leq N)} = 90\%$, this will be an approximate value $\mathrm{AAS} \doteq 96{,}189\ \mathrm{Mb/s}$. The utilisation factor UF further specifies the nature of network utilisation by end-users, i.e. not in the maximum possible way, but in a typical way for a given area and type of data traffic. For example, for the value of utilisation factor $\mathrm{UF} = 0{,}421$ with average bit rate $\mathrm{NBR_{avg}} = 127{,}3\ \mathrm{Mb/s}$ (according to network traffic monitoring), this will be approximate value $\mathrm{AAS} \doteq 164{,}329\ \mathrm{Mb/s}$.

IV. Terms, definitions and abbreviations

B (*bandwidth*) — general designation of bandwidth of transmission medium and technology in Hz

NAS — normally available speed that the end-user can assume and actually achieve when downloading and uploading data at a time equivalent to 95% of the time of 1 calendar day

CAP (*link capacity)*— refers to the total capacity of a given data link in a given direction (uplink or downlink). The capacity is mostly related to the physical possibilities of a given data link, or to the set limit values (actual bandwidth in the given direction)

CAP' (*link adjusted capacity*) — the theoretically adjusted value of the given data link capacity due to the impact of the value of the utilisation factor UF

$\mathrm{CE_{thr}}$ (*capacity expansion threshold*) — capacity expansion parameter in the case of mobile network technology, for 5G networks, the typical and at the same time, the maximum recommended value is 70 %

DeP x (*demarcation point x*) — refers to a specific demarcation point as a transfer interface between two distinct network entities (backbone network, access network, local network, etc.)

DP (*distribution point)* — distribution point (node) of the distribution network belonging to the access network set

$\mathrm{F(t)}$ — distribution function of a given probabilistic distribution

FS (*frame size*) — size of Ethernet frame

FTD (*frame time delay)*— delay in frames representing the time delay NUT between sending and receiving of the Ethernet framework

$\mathrm{FTD}\ (RTT)$ — corresponds to the elapsed period between sending of the first bit of the frame from the end-user to the internet access service provider and the receipt of the last bit of the returned frame from the service provider to the end-user

FLR (*frame loss ratio)* — Frame loss as the ratio of all non-delivered (lost) Ethernet frames to the total number of all ethernet frames sent

FPS (*frame per second*) — Maximum number of frames on ISO/OSI model connection layer

FWA (*fixed wireless access*) — wireless electronic communications network at a fixed location

IFDV (*inter-frame delay variation)* — the inter-frame delay variation, often also the delay variation or jitter, represents the difference between the Ethernet frame delivery reference time ($\mathrm{c_{k}}$) and its actual delivery time ($\mathrm{d_k}$)

IPER (*IP packet error ratio)* — the value of packet error ratio in the form of the ratio of wrongly received packets to all received packets.

IR (*information rate*) — value of the information rate corresponding to the ISO/OSI model connection layer

k — number of data flows

L x (*layer x*) — specific ISO/OSI model layer

LU (*link utilisation*) — refers to the use of a given data link in the form of a percentage utilisation of the total capacity of a given data link

$\mathrm{LU}_{\mathrm{max}}$ — refers to the maximum utilisation value of a given data link LU, which may correspond to the capacity value CAP

N — general indication of the number of termination points (NTP) in mathematical relationships

NTP (*network termination point*) — network termination point meeting the criteria specified in BEREC Guidelines BoR (20) 46

MTU (*maximum transmission unit*) — designation for the maximum size of IP datagram (TCP segment) that can be sent by a given network interface

NBR (*net bit rate*) — refers to the transmission speed, often referred to as bit rate, corresponding to the physical layer of the ISO/OSI model of the interface with the assumption of the ethernet framework utilisation

$\mathrm{NBR}_{\mathrm{avg}}$ — average bit rate NBR over the monitored timeframe (typically from network traffic monitoring)

$\mathrm{NBR}_{\mathrm{max}}$ — maximum bit rate NBR over the monitored timeframe (typically from network traffic monitoring)

$\mathrm{p}_{\mathrm{k}}(\mathrm{t})$ — probability of the existence of k data flows in an electronic communications network (on the assessed data link) over time t

$\mathrm{P}_{\mathrm{SRD}}$ — the decrease in AAS value is determined as the ratio of variance σ_{AAS} and maximum value $\mathrm{AAS}_{\mathrm{max}}$, the decrease is often reported in [%]

AAS — Actually achieved speed, i.e., the TCP throughput over a given measurement period corresponding to the actual performance of the service

$\mathrm{AAS}_{\mathrm{max}}$ — maximum value AAS, i.e., TCP permeability, over a given time interval

$\mathrm{AAS}_{\mathrm{min}}$ — minimum value AAS, i.e., TCP permeability, over a given time interval

$\mathrm{TCP\ TR}$ (*TCP throughput)*— the transmission speed corresponding to the transport layer of the ISO/OSI model using the TCP protocol (link-oriented)

$\mathrm{TR}_{\mathrm{DL}}$ (*downlink/uplink time ratio)*— the ratio of time utilisation of individual directions of data communication, e.g., 75 % in the downlink direction (25% in the uplink direction)

UF (*utilisation factor*) — The utilisation factor corresponds to the average speed in peak time $(\mathrm{NBR}_{\mathrm{max}})$ and average speed $(\mathrm{NBR}_{\mathrm{avg}})$ over the monitored timeframe (e.g., month)

t — time [s]

$\Delta\mathrm{t}$ — time interval [s]

κ *(adjustment factor)*— the adjustment factor determining the value CAP' against the original value CAP, the value of the factor corresponds to the ratio of the original capacity value CAP and the utilisation factor UF with the value of the average bit rate $\mathrm{NBR}_{\mathrm{avg}}$, or the inverted value of the data link utilisation LU

η (*spectrum efficiency*) — spectral efficiency of mobile network technology, with a typical value of 17 b/s/Hz in the case of 5G networks

σ_{AAS} — variance of AAS value determined as the difference of values $\mathrm{AAS}_{\mathrm{max}}$ and $\mathrm{AAS}_{\mathrm{min}}$ in [Mb/s]

λ — mean input intensity of Poisson probability distribution

5G — designation of mobile network technology